MATH SKILL INFORMATION PAGE

Algebra	For use with 13-3

Solve a Quadratic by Completing the Square

PURPOSE: Completing the square when you see a quadratic will allow you to solve the quadratic as you learned to do in the last section (13-2). First you have to get the quadratic into the $(x - \quad)^2$ form.

The **following four examples illustrate how to solve a quadratic by completing the square**, going from the easiest example to the hardest. If the textbook seems confusing, see if these step-by-step examples help you better understand how to do this.

EXAMPLE 1: Complete a Square *(Change $x^2 + bx$ into $(x - \quad)^2$)*

Example 1: $x^2 - 6x$

1. **Figure out the new "c"**, which will make a perfect square inside the parentheses.

   $c = (b/2)^2 = (-6/2)^2 = (-3)^2 = 9$

   MEMORIZE THIS: $c = \left(\frac{b}{2}\right)^2$

2. **Put that new c after the bx.** $x^2 - 6x + 9$

3. **Do the square**: $(x - 3)^2$

4. You are going to use that square in the next problem, where you have to solve an equation. So, it's important for you to realize that this skill of completing the square is one part of bigger problems, as examples 2 and 3 show in section 13-3.

NOTE: You see in this example that the coefficient of x^2 is one. That's really important: you can ONLY complete the square if the coefficient of x^2 is one. So $x^2 - 6x$ is possible for completing the square. However, something like $3x^2 - 18x$ cannot be made into a square—unless you get rid of that coefficient of the x^2, the 3. Here's how you do that:

a. Factor that coefficient from BOTH terms: $3(x^2 - 6x)$
b. Now work with just the inside part, and complete the square for that part—since the coefficient of the x^2 that is inside the parentheses is 1. So the new c = 9, and you now have $3(x^2 - 6x + 9)$. Notice that you have to keep the 3 in front of the parentheses.
c. Now do the square: $3(x - 3)^2$. Notice that you have to keep the 3 in front of the square.
d. IMPORTANT: in example 4 in this handout, you have to do this type of factoring before completing the square. Once you complete the square, you have to also add a corresponding amount of the new c to the other side ("if you do something to one side of an equation, you MUST do the same exact thing to the other side of the equation"). What this means, for $3(x^2 - 6x + 9)$, is that you really didn't add just 9; you actually added 27, because that 3 has to multiply to everything inside the parentheses. So you'll have to add 27 to the other side, to account for 3 times 9, not just 9.

EXAMPLE 2: (When you get perfect squares on both sides of the equation.)

Solve a Quadratic by Completing the Square

Change $x^2 + bx + c = 0$ *into* $(x - \quad)^2 = ?$, *so that you can solve* $(x = \_\_\_)$

Example 2: $x^2 - 6x - 16 = 0$
1. **Move the c** to the other side of the equal sign. $x^2 - 6x \qquad = 16$
3. **Figure out the new "c"**, which will make a perfect square inside the parentheses. $c = (b/2)^2 = (-6/2)^2 = (-3)^2 = 9$
4. **Put that new c into the equation.** BUT you have to put it into TWO places: • First, you have to put it into the parentheses, so that it can make a perfect square with the x^2 and the $-6x$. That makes: $x^2 - 6x + 9 = 16 + \ldots\ldots..$ • Second, you have to add it to the other side, so that the net result is zero. $x^2 - 6x + 9 = 16 + 9$ After adding on the right, you have $x^2 - 6x + 9 = 25$
5. **Do the square**: $(x - 3)^2 = 25$ NOTE: Remember that the operation inside the parentheses must be the same as the original "b." In this case, the "b" is negative, so you must put a minus sign inside the parentheses.
6. Now do what you learned in section 13-2: **a. Take the square root of both sides:** $(x - 3)^2 = 25 \Rightarrow \sqrt{(x-3)^2} = \pm\sqrt{25}$ **b. Simplify both sides:** $x - 3 = \pm 5$ **c. Isolate the x:** $x - 3 + 3 = 3 \pm 5$ **d. Do the separate solutions:** $x = 3 + 5$, and $x = 3 - 5$. **e. The solutions are:** $x = 8$, and $x = -2$. Those two numbers will be x when the y is zero.

EXAMPLE 3: (When you do **not** get a perfect square on the right side.)

Solve a Quadratic by Completing the Square

Change $x^2 + bx + c = 0$ *into* $(x - \quad)^2 = ?$, *so that you can solve* $(x = \_\_\_)$

Example 3: $x^2 - 8x - 3 = 0$

1. **Move the c** to the other side of the equal sign.

   $x^2 - 8x = 3$

3. **Figure out the new "c"**, which will make a perfect square inside the parentheses.

   $c = (b/2)^2 = (-8/2)^2 = (-4)^2 = 16$

4. **Put that new c into the equation.** BUT you have to put it into TWO places:
   - First, you have to put it into the parentheses, so that it can make a perfect square with the x^2 and the $-8x$. That makes: $x^2 - 8x + 16 = 3 + \ldots\ldots$
   - Second, you have to add it to the other side, so that the net result is zero.
     $x^2 - 8x + 16 = 3 + 16$ After adding on the right, you have $x^2 - 8x + 16 = 19$

5. **Do the square**: $(x - 4)^2 = 19$

   NOTE: Remember that the operation inside the parentheses must be the same as the original "b." In this case, the "b" is negative, so you must put a minus sign inside the parentheses.

6. Now do what you learned in section 13-2:

   a. Take the square root of both sides: $(x - 4)^2 = 19 \Rightarrow \sqrt{(x-4)^2} = \pm\sqrt{19}$

   b. Simplify both sides: $x - 4 = \pm\sqrt{19}$ (See the * below this example.)

   c. Isolate the x: $x - 4 + 5 = 4 \pm \sqrt{19}$

   d. Do the separate solutions: $x = 4 + \sqrt{19}$, and $x = 4 - \sqrt{19}$.

* In this case, $\sqrt{19}$ cannot be simplified. However, sometimes you have a square root that can be simplified, such as $\sqrt{12}$. Here's how to simplify something like that:

1. Rewrite $\sqrt{12}$ as $\sqrt{4 \cdot 3}$.
2. Use the rules of square roots to separate the parts: $\sqrt{4} \cdot \sqrt{3}$
3. Since $\sqrt{4} = 2$, you replace the $\sqrt{4}$ with the 2, to make $2\sqrt{3}$ You don't need to show • for multiplying, since you treat the square root like a variable (like 2x; you don't need •)
4. So $\sqrt{19} = 2\sqrt{3}$.

SPECIAL NOTE FOR ALGEBRA STUDENTS IN MONTH 8

Simplifying square roots is taught in chapter 11, which we have not yet covered. We will go back to chapter 11 next month (month 9). Until then, I will not hold you accountable for simplifying square roots. You may automatically leave a square root as it is, even if it's something like $\sqrt{12}$, and could be simplified. You may simplify if you want, **if you can do it correctly**, but you don't have to in month 8 — but **ONLY** in month 8. After learning how to do it better in month 9, you will **ALWAYS** have to simplify square roots when possible, just like reducing fractions.

EXAMPLE 4: (When the coefficient of the x^2 term is not 1.)

Solve a Quadratic by Completing the Square

Change $ax^2 + bx + c = 0$ *into* $a(x - \quad)^2 = ?$, *so that you can solve* $(x = \_\_\_)$

Example 4: $2x^2 - 10x - 17 = 0$
1. **Move the c** to the other side of the equal sign. $2x^2 - 10x \qquad = 17$
2. **Remove the "a"** from the first term. NOTE: You ALSO have to factor that "a" from the second term. $2(x^2 - 5x \qquad) = 17$ Consider the removal in this way: $\frac{2x^2}{2} - \frac{10x}{2} = x^2 - 5$
3. **Figure out the new "c"**, which will make a perfect square inside the parentheses. $c = (b/2)^2 = (-5/2)^2 = 25/4$
4. **Put that new c into the equation.** BUT you have to put it into TWO places: • First, you have to put it into the parentheses, so that it can make a perfect square with the x^2 and the $-5x$. That makes: $2(x^2 - 5x + 25/4) = 17 + \ldots\ldots$ • Second, you have to add it to the other side, so that the net result is zero. NOTE: if there's an "a" outside the parentheses, then you must multiply by that "a" in order to get the correct amount that needs to be added. So, $2 \bullet 25/4 = 25/2$. $2(x^2 - 5x + 25/4) = 17 + 25/2$
5. **Do the square**: $2(x - 5/2)^2 = 17 + 25/2$ NOTE: Remember that the operation inside the parentheses must be the same as the original "b." In this case, the "b" is negative, so you must put a minus sign inside the parentheses.
6. **Do the end operation**: $17 + 25/2$. This is adding fractions, so the denominators must be the same. $\frac{17}{1} + \frac{25}{2} = \frac{17}{1} \bullet \frac{2}{2} + \frac{25}{2} = \frac{34}{2} + \frac{25}{2} = \frac{59}{2}$
The equation now looks like: $2(x - 5/2)^2 = 59/2$ Now you have to solve for the x, which means to isolate it.
7. **Divide by the number outside the parentheses**. In this case, it's 2. $(x - 5/2)^2 = 59/2 / 2$ $59/2 / 2$ is the same as $(59/2) \div 2$, which is $(59/2)\bullet(1/2)$, which is $59/4$. So, $(x - 5/2)^2 = 59/4$
8. **Take the square root of both sides**. $(x - 5/2)^2 = 59/4 \Rightarrow x - 5/2 = \pm\sqrt{59/4}$ The square root of 4 is 2, so it's really $\pm\sqrt{59}\ /2$. That makes the equation $x - 5/2 = \pm\sqrt{59}\ /2$.
9. **Isolate the x**. $x - 5/2 = \pm\sqrt{59}\ /2 \Rightarrow x = 5/2 \pm \sqrt{59}\ /2$.
In this example, the solutions are: $x = 5/2 + \sqrt{59}/2$ and $5/2 - \sqrt{59}/2$.
10. ***Compute the two solutions****. Do this if you don't have square roots in the answer, such as:* $x = -14 \pm\sqrt{36} = -14 \pm 6$ *which is* $x = -14 + 6$ *AND* $-14 - 6$, *so* $x = -8, -20$.