

THE COIN PROBLEM

These types of problems require making two equations, and then solving the equations by substitution or addition. One equation usually comes from adding up the number of coins (irregardless of the value of the coins), and the other equation uses the value of the coins.

Example: Kami has some nickels and some dimes. The value of the coins is \$1.65. There are 12 more nickels than dimes. How many of each kind of coin does Kami have?

Step 1: Figure out what information you've been given, and what you're supposed to find.

- We know that the value of the coins is \$1.65.
- We know that the coins are nickels and dimes.
- We know that there are more nickels than there are dimes—12 more nickels, to be exact.
- We need to figure out how nickels Kami has, and how many dimes she has.

Step 2: Make the first equation.

- This equation will make the number (not the value) of nickels equal to the number of dimes. The fact that nickels are worth less than dimes doesn't matter yet; only the number of actual coins matters.
- Let's use n for nickels, and d for dimes.
- We can't just make $n = d$. We know there are more nickels than dimes. BE CAREFUL: we can NOT put $n + 12 = d$. That would be saying there are more dimes than nickels—think about it.
- We have to say $n = d + 12$. Or, we could say $n - 12 = d$. Both mean the same thing: we have to add 12 to the dimes to get the same number as the nickels, or we have to subtract 12 from the nickels to get the same number as the dimes. Your choice which way to put it. Today we'll use $n = d + 12$.

Step 3: Make the second equation.

- This equation will use the **value** of the coins. This is where it is now important that nickels are worth 5 cents and dimes are worth ten cents.
- The value of a nickel is \$0.05. If you had 16 nickels, the value would be \$0.05 times 16. So, if you have n nickels, the value is \$0.05 times n , or $0.05n$.
- The value of a dime is \$0.10. If you had 27 dimes, the value would be \$0.10 times 27. So, if you have d dimes, the value is \$0.10 times d , or $0.10d$.
- We know that the value of all the coins comes to \$1.65. We can say that **the value of all nickels plus the value of all dimes equals the total value**.
- That sentence, put into an equation, is: $.05n + .10d = 1.65$. It's often easier to work with integers than with decimals, so remove decimals by multiplying by a factor of 10. 100 is the best one this time, because there are two decimal places in all the numbers. Thus: $5n + 10d = 165$.

Step 4. Solve.

- You have two equations, $n = d + 12$ and $5n + 10d = 165$.
- Use either the substitution or addition method. For this example, we'll use the substitution method.
- Since n is the same as $d + 12$, we can put $d + 12$ in place of n in the equation, $5n + 10d = 165$.
- So, $5n + 10d = 165$ becomes $5(d + 12) + 10d = 165$. Since there is only one variable now, we can solve this equation for that one variable, d , and we'll then know how many dimes Kami has.
 $5(d + 12) + 10d = 165$
 $5d + 60 + 10d = 165$
 $15d = 105$, so $d = 7$
- The d stands for dimes. If there are 7 dimes, there must be 19 nickels (Since $n = d + 12$, $n = 7 + 12$: $n = 19$).
- If this problem had started as a system of equations, we would show the answer in coordinate-point form (x,y) , or $(19, 7)$. However, the system of equations was simply a method for solving the original question, which was how many nickels and dimes Kami has.
- The answer: **Kami has 19 nickels and 9 dimes.**