THE DIGIT PROBLEM

These types of problems require making two equations, and then solving the equations by substitution or addition. One equation usually comes from adding the amount of the digits, and the other equation uses the place value of the digits.

Example: The sum of the digits of a two-digit number is 14. If the digits are reversed, the number is 36 greater than the original number. Find the original number.

Step 1: Figure out what information you've been given, and what you're supposed to find.

- We know that we're using a two-digit number.
- We know that if we separate the two digits and then add them, we get 14.
- We know that if we reverse the digits, the new number is 36 larger than the original number.
- We need to figure out the original number.

Step 2: Make the first equation.

- Make variables for the two digits: x and y. So the number is xy. That means the x is the number that is in the ten's place, and the y is the number in the one's place.
- We know if we add the two digits, we get 14. Adding the two digits looks like this: $\boldsymbol{x + y = 14}$.

Step 3: Make the second equation.

- Now we care about the actual value of the number. The number in the ten's place is actually that number times 10. The number in the one's place is actually that number times one. Example: 83 is actually 80 + 3, which is actually 8 times 10 plus 3 times 1.
- So the value of the x is really x times 10 (or, $10x$). The value of the y is really y times 1 (or, $1y$, or y).
- That means $xy = 10x + y$, but that's not the equation that will help us. The fact that xy is really $10x + y$ will definitely be useful next.
- Now we use the information about reversing the digits. When we don't reverse the digits, the number we have is really $10x + y$. If we reverse the digits, the y goes into the ten's place, and the x goes into the one's place. That means the new number is really $10y + x$.
- Our information about the number with the reversed digits is that it is 36 larger than the first number. Here's a way to look at it:

The original number	plus	36	is equal to	the new number.
$10x + y$	+	36	=	$10y + x$

- When we simplify that equation, we get $\boldsymbol{9x - 9y = -36}$. It's more useful to move the 36 to the right.

Step 4. Solve.

- You have two equations $x + y = 14$, and $9x - 9y = -36$.
- Use either the substitution or addition method. For this example, we'll use the addition method.
- Multiply equation 1 ($x + y = 14$) by 9. This will allow us to cancel out the y terms in both equations when we add them. So, equation 1 now becomes $9x + 9y = 126$
- Add:
$$\begin{array}{r} 9x + 9y = 126 \\ \underline{9x - 9y = -36} \\ 18x = 90 \end{array}$$
- Continue solving: $x = 5$.
- The x stands for the tens digit in the original number. The first equation was $x + y = 14$. Since x is 5, the equation becomes $5 + y = 14$, and therefore y must be 9

Step 5. Put the answer in the correct form, if necessary.

- If this problem had started as a system of equations, we would show the answer in coordinate-point form (x,y), or (5, 9). However, the system of equations was simply a method to find the original number.
- The answer: **the original number is 59**. Check: $5 + 9 = 14$. $95 = 36 + 59$.