INTERMEDIATE ALGEBRA

Section 10-2: Logarithms and Logarithmic Functions

PART 1: CHANGING FORMS

1. From Logarithmic to Exponential, and Back Again. *(Examples 1 and 2)*
   a. This method essentially allows you to remove the work "log" from an equation. That becomes very helpful when solving equations later. The meaning, though, has to do with finding the correct power used to raise a number to get a different number.
   b. In Example 1a, you see that you rewrite the equation by simply moving things around.
      i. You read the original as "log base 8 of 1 equals 0" ($\log_8 1 = 0$).
      ii. When changed, you read it as "1 equals 8 raised to the power of 0" ($1 = 8^0$).
   c. Example 1b does the same:
      $\log_2(\mathbf{1/16}) = \mathbf{-4}$ becomes $(\mathbf{1/16}) = \mathbf{2^{-4}}$.
   d. Example 2a and 2b show that you can go the other direction:
      $\mathbf{10^3 = 1000}$ is the same as $\mathbf{\log_{10}1000 = 3}$.

2. Inverse Property *(Example 4)*
   a. Follow the bottom of page 532 and top of page 533: Since the basic exponential and logarithmic equations are inverses of each other, they "undo" each other (top of p. 533). Knowing this allows you to essentially cancel them out, when you see them in the forms shown in example 4.
   b. Look at Example 4a, and follow these steps, which are explained in Part 2, 1. below.
      i. $\mathbf{\log_6 6^8 = x}$
      ii. $\mathbf{6^x = 6^8}$
      iii. $\mathbf{x = 8}$
      iv. $\mathbf{\log_6 6^8 = 8}$, because you can now substitute the 8 for x in the first step.
      v. You can now generalize this: **whenever you see $\mathbf{\log_b b^y}$, that is automatically y**.
   c. Example 4b shows you to generalize that **whenever you see the format $\mathbf{b^{\log_b y}}$, that is automatically y**.

3. **LEARN/MEMORIZE**:
   - You can change these equations both ways: from log to exponent, from exponent to log.
   - Pay attention to the details of where the base, the number, and the exponent all go to and come from.
   - When you see the formats $\log_b b^y$ and $b^{\log_b y}$, both are simply equal to y.

PART 2: EVALUATE AND SOLVE LOGARITHMIC EXPRESSIONS AND EQUATIONS *(Examples 3 and 5)*

1. Use "format change" to get an answer. Look at example 3: "**Evaluate $\mathbf{\log_2 64}$.**"
   a. You have to make the expression into an equation by setting it equal to a variable. So now you have $\mathbf{\log_2 64 = x}$ (I prefer using x instead of the textbook's choice of y).
   b. "Format change" allows you to rewrite $\mathbf{\log_2 64 = x}$ as $\mathbf{2^x = 64}$.

c. Now you use the Property of Equality for Exponential Functions, which means that if you have the bases in an equation are equal, then the exponents are also equal.
d. But the 2 and 64 are not equal! Oh, wait a moment—yes, they are. You just have to rewrite the 64 as a power of 2. Here is a place where, if you don't recognize that 64 is a power of 2, you need to play with numbers (and your calculator) and see what you come up with. $2^4 = 16$, $2^5 = 32$, $2^6 = 64$…ahh, there it is. Use 2^6 in place of 64.
e. So now, **instead of $2^x = 64$, we have $2^x = 2^6$**. Since the bases are the same, the exponents must be the same, and **$x = 6$**.
f. Here's an exponential equation example to illustrate a number of concepts: $\mathbf{3^{5x+28} = 9^{x+2}}$.
    i. $\mathbf{3^{5x+28} = (3^2)^{x+2}}$ The 9 can be written as 3^2.
    ii. $\mathbf{3^{5x+28} = 3^{2(x+2)}}$ The rules of exponents make you multiply the 2 and the x+2.
    iii. $\mathbf{3^{5x+28} = 3^{2x+4}}$ The bases are the same, so the exponents are equal.
    iv. $\mathbf{5x + 28 = 2x + 4}$, which can be simplified to $\mathbf{3x = -24}$. Therefore, $\mathbf{x = -8}$.
g. INEQUALITIES: Solve them in the same way as you solve the equations. Since you don't have a variable in the denominator, you don't have make a number line and test the regions.

2. **LEARN/MEMORIZE**:
   - Use "format change" to solve log equations/inequalities.
   - Since the bases must match, remember to rewrite numbers in order to have the same base.

<u>PART 3</u>: EVALUATE AND SOLVE LOGARITHMIC INEQUALITIES *(Example 6)*

1. You have two cases:
   a. Greater Than: $\log_b x > y$
   b. Less Than: $\log_b x < y$
2. If **<u>greater than</u>**,
   a. change the logarithmic equation into an exponential equation *(follow Part 1, 1b and 1c above)*,
   b. then solve like normal.
3. If **<u>less than</u>**,
   a. change the logarithmic equation into an exponential equation *(follow Part 1, 1b and 1c above)*,
   b. x must ALSO be greater than 0, so you must put 0 < with your inequality,
   c. then solve like normal.

<u>PART 4</u>: LOGARITHMS ON BOTH SIDES *(Examples 7 and 8)*

1. Notice in both examples 7 and 8 on page 534, if you have the same log base on both sides, then the numbers must be equal (in practical terms, the logs cancel out).
2. Example 8 illustrates one extra thing: logs must be greater than 0. The implication of that is when you get a solution, you must check the solution, and be sure that it doesn't make one of the logarithms zero or less than zero.