INTERMEDIATE ALGEBRA
Notes on Chapter 6

These notes do not in any way replace your own work and study (and these are NOT the notes that you can turn in for extra credit—see the guidebook for information on how to correctly make those notes). These are important points and hints that can help you avoid some of the common mistakes that students tend to make in these sections.

SECTION 1: GRAPHING QUADRATIC FUNCTIONS

1. Know the Key Concept Box, p. 287.
   a. Identifying the three things (y-int, eq of axis of symmetry, and x-coordinate of the vertex) depends on knowing a, b, and c, so you must always put a quadratic function into the form $\mathbf{y = ax^2 + bx + c}$. a can not equal 0 because then you'd have a straight line, not a parabola (which is what a quadratic equation describes).
   b. Memorize especially that **–b/2a**, because you get lots of use out of it.
   c. **NOTE:** The "equation of the axis of symmetry" is always in the format: x = ___ (that's what an equation is; it has an equal sign).
   d. Memorize how these things look on a graph (i.e., the model in the Key Concept, p. 287).
2. **Maximum and minimum values**: don't get them mixed up. See the models in the Key Concept box, p. 288.
   a. A parabola has a maximum value when a is negative, such as $y = -2x^2 + 3x + 4$. That means that it opens down, so the vertex is the highest point.
   b. A parabola has a minimum value when a is positive, such as $y = 2x^2 + 3x + 4$. That means that it opens up, so the vertex is the lowest point.
   c. The **y-coordinate** of the vertex is the actual value of the maximum or minimum value.

SECTION 2: SOLVING QUADRATIC EQUATIONS BY GRAPHING

1. Solutions = roots = zeros. These are the places where the parabola touches or crosses the x-axis. The x-coordinates of those points are the solutions/roots/zeros. They are the zeros because the y-value of the coordinates are 0, since they're on the x-axis, the line y = 0.
2. Note the differences for 1 real solution, 2 real solutions, and no real solutions (top of 295).
3. When exact roots can't be found by graphing, you can estimate roots. Look closely at example 4 on page 296. Use integers in the x/y table, and look for the places where the signs on the y values change. The corresponding x values are bracketing the zeros, and rather than give a decimal answer (such as 0.2679 and 3.7320 as would be the solutions for example 4), you will give "between 0 and 1 and between 3 and 4" when asked to estimate the roots.
4. **GRAPHING TIPS:**
   a. Find the vertex: –b/2a for the x-coordinate, substitute it in the equation for the y-coordinate.
   b. Make the x/y table. Put the vertex in the middle, and at least 2 integers on either side of the x-value.
   c. Graph the 5 coordinates. Check yourself with opening up or down (a is positive or negative), and making sure the y-intercept is correct (the c).

SECTION 3: SOLVING QUADRATIC EQUATIONS BY FACTORING

1. The **Zero Product Property** is the mathematical concept behind solving. If two things multiply to make zero, then at least one of those two things must be zero (there's no other way to multiply to get zero).
2. **WARNING**: Put all terms on one side, so it all = 0. Look at example 1, p. 301, and also pay close attention to problem #3 on page 303. Don't make Lina's error!
3. **Note the difference between factored and solved.** In example 1, the factorization of $x^2 = 6x$ is $x(x - 6)$, but the solutions are {0, 6}. Note that when you do $x - 6 = 0$, you get $x = 6$, so when you see solutions, you can go backwards and use them to make the factors, as long as you use the opposite sign (such as –6 for 6, in this example).
4. Continuing this thought, if you're given roots and are told to **write an equation**, just plug them into the factored form—but change the signs! Closely study example 4 on page 303. You set up (   )(   ) = 0 and then multiply in order to put it into standard form. NOTE that you don't leave fractions, if any come up. Multiply by the LCD as the example shows (and on the zero side, the LCD goes away).
5. Notice what a **double root** (example 2) means: the parabola has moved so that the two x-intercepts have now come together at the same point. The parabola now just touches the x-axis, and doesn't go cross it. We don't say the number twice, we only use it once, but it's obvious, especially from a graph, that the single solution signifies a double root.

SECTION 4: COMPLETING THE SQUARE

1. Don't forget to put ± (plus or minus) whenever you use the square root property. That's because any square root can be negative, since two negatives multiply to a positive. If you don't put the ±, you won't end up with the two answers you need.
2. When you have a radical in an answer (such as $\sqrt{3}$) you don't always need to use a calculator; you can leave it in that form. ALWAYS simplify, though; $\sqrt{12}$ is never correct; $2\sqrt{3}$ is.
3. Notice in examples 4, 5, and 6, the original c value was moved out of the way (in these cases by moving it to the other side of the equation). Don't forget to do this!
4. Memorize and practice using $(b/2)^2$ to get the c you need.
5. **WARNING 1:** if the "a" is not 1 (the coefficient with the x^2 term), then you must divide it throughout. Yes, you'll have a fraction for b, but that's necessary. If you're shaky on fractions, now you have to get better at them. If you need to, come to see your teacher to learn a couple of ways to handle fractions in these types of problems.
6. **WARNING 2:** after getting the c, you MUST remember to also put that same amount on the other side of the equation.

SECTION 5: THE QUADRATIC FORMULA AND THE DISCRIMINANT

1. **WARNING 1:** Memorize the quadratic formula. Don't argue, don't complain, don't put it off. If you don't know it cold, you will lose lots of points this year and in any future math class you ever take. And make sure you get the little parts exactly right: the negative b, the plus or minus, the b^2, the minus 4ac, the 2a. Know it perfectly, be able to write it out anytime.
2. **WARNING 2:** If you're shaky on using negatives, this it where you need to get better—now. Most of the mistakes people make with the quadratic formula involves a negative sign.
3. Know your square roots—memorize the first 12, at least. You can use your calculator, but you'll save yourself a LOT of time if you simply know them cold: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144…even 169, 196, 225 will be very helpful.
4. Make sure you also remember ***i*** (see example 4, page 315).
5. **THE DISCRIMINANT**: That part of the quadratic formula that's under the radical sign can, by itself, give you enough information for you to know how many and what types of roots the

original quadratic equation has. Here's a short summary:

If the discriminant is:	there are:
positive, AND a perfect square,	2 real, rational roots.
positive, but NOT a perfect square,	2 real, irrational roots.
equal to zero,	1 real, rational root.
negative,	2 complex roots.

6. CONCEPT SUMMARY on page 317 is important to help you understand the purpose of all five methods for solving quadratic equations. Notice you can always use the quadratic formula. However, you have to know all five methods well enough to use them on the test!

SECTION 6: ANALYZING GRAPHS OF QUADRATIC FUNCTIONS

1. **WARNING:** In the vertex form of a quadratic equation [$\mathbf{y = a(x - h) + k}$] the *h* is the OPPOSITE of the number with the x. Using $y = 4(x + 7)^2 + 5$ as an example:
   a. The *h* (the x-coordinate of the vertex) is –7. Don't forget to take the opposite sign.
   b. For the *k* (the y-coordinate of the vertex), you DO NOT take the opposite sign. It's simply whatever it is. In this case, the *k* is 5.
   c. The vertex (*h*, *k*) is therefore (–7, 5) in this case. Notice that the *a* does not affect the h and k (yet).
2. Pay close attention to the **variations** of a parabola.
   a. The *k* tells you if the parabola should be moved up or down,
   b. the *h* tells you if it should be moved right or left,
   c. the sign of the *a* (positive or negative) tells you what way the parabola opens up,
   d. the *a* itself tells you how wide the parabola opens.
3. **CHANGING FROM STANDARD FORM TO VERTEX FORM:**
   a. If it's not already a perfect square, you need to make it one. You do that by moving the *c* out of the way, then completing the square.
   b. **WARNING 1:** Don't forget to account for the fact that you're adding something new when you complete the square. Look at example 2, p. 324. Since you add 16 to complete the square, you must also subtract 16, in order to keep the equation equal. Back in section 4 you put the new thing on each side, but here you can't do that, because you must keep y alone on the left side.
   c. **WARNING 2:** If the "a" is not 1 (the coefficient with the x^2 term), you must factor it out of JUST the $ax^2 + bx$ terms (remember that you move the *c* off to the side while you're completing a square). COMMON ERROR: people forget to divide the *b* by the *a* when factoring out the *a*.
   d. **WARNING 3:** After you complete the square, the new amount must be added to the c that you moved out of the way. COMMON ERROR: people forget to FIRST multiply the new amount by the *a* that they first factored out of the $ax^2 + bx$ terms. See example 3 on page 324 on exactly how to handle this.
4. Writing an equation when given points: Notice in example 4 (page 325) that you use the vertex and the point to find the *a*. Then you write the vertex form with the *a*, *h*, and *k*. **NOTE:** leave that extra point out of the final equation; you only needed it to help you find the *a*.

SECTION 7: Make your own list of important things to remember.