# INTERMEDIATE ALGEBRA

## Chapter 8 Notes and Information

## 8.1 MIDPOINT AND DISTANCE FORMULAS

Memorize these, carefully and exactly.

- **Distance Formula**: $d = \sqrt{(x_2 - x_1)^2 + (x_2 - x_1)^2}$
- **Midpoint Formula**: $M = ( (x_1 + x_2)/2 , (y_1 + y_2)/2 )$

## 8-2 PARABOLAS

- **Conic sections** (see sliced double cones on pages 419 and 449).
- **Parabola**: Set of all points in a plane that are the same distance from the focus and directrix (see picture near bottom of page 419).

**"*SOLVE A PARABOLA*"** means to use the standard form of the parabola to graph it and find all the information below. You can graph a parabola by using an x-y chart or the information below.

1. **Parabola equation**: Standard form: $y = ax^2 + bx + c$. You might have to complete the square in order to put the equation into standard form.
   a. Standard form for up or down is: $y = a(x - h)^2 + k$.
   b. Standard form for right or left is: $x = a(y - k)^2 + h$.
2. **Vertex**: (h, k). The central point on the line of the parabola. Use the standard form to find h and k. **HINT**: the sign is opposite of the k in the equation but the same as the k.
3. **Axis of symmetry**: The axis that intersects the parabola at the vertex, and about which the parabola could rotate. (This one is easy: just use h or k, but **you must use x =, or y =**.)
   a. Up or down: $x = h$.
   b. Right or left: $y = k$.
4. **Focus**: A specific given point inside the parabola.
   a. Up or down: $(h, k + 1/4a)$
   b. Right or left: $(h + 1/4a, k)$
5. **Directrix**: A given line outside the parabola.
   a. Up or down: $y = k - 1/4a$
   b. Right or left: $y = h - 1/4a$
6. **Direction of opening**:
   a. +a, the parabola opens up (or right).
   b. –a, the parabola opens down (or left).
7. **Length of latus rectum**: $|1/a|$ units. This is a line segment that is perpendicular to the axis of symmetry, goes through the focus, and the end points lie on the parabola. See the picture on the bottom of page 421.

# 8-3 CIRCLES

1. Basic Information:
    a. **Circle**: Set of all points in a plane that are the same distance from a given point in the plane, called the center (see picture in the middle of page 426).
    b. **Circle equation**: Standard form: $(x - h)^2 + (y - k)^2 = r^2$. You might have to complete the square in order to put the equation into standard form.
    c. **Center**: (h, k). Find h and k in the standard form equation. **NOTE**: the signs change.
    d. **Radius**: r.
2. Write an equation when given a diameter (see example 2, page 427):
    a. Use the midpoint formula to find the center.
    b. Use the distance formula to find the radius (using either of the two diameter endpoints).
    c. Put the center (h, k) and the radius (r) into the circle's standard equation form.
3. Graph a circle:
    a. When the equation is in standard form: you have the center and radius, so plot some points and draw the circle through them, with the center in the middle (see ex. 4, p 428).
    b. When the equation is not in standard form, you first have to complete the squares for both the $x^2$ and $y^2$ quadratics, in order to have h and k for the center. Be careful when completing the squares; remember that you have to add both new c's to the right side of the equation also, after first moving the constant to that right side, if it's not already there. (See example 5, page 428.)

# 8-4 ELLIPSES

- **Ellipse**: Set of all points in a plane such that the sum of the distances from two fixed points is constant. The fixed points are called foci. (See picture in the middle of page 433).

1. **Ellipse equation**: Standard form: You might have to complete the square in order to put the equation into standard form.

| a. Longer axis up and down: | b. Longer axis right and left: |
|---|---|
| $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ | $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ |

2. **Center**: (h, k). The central point of the ellipse.
    a. Use the standard form to find h and k.
    b. **HINT**: the sign is opposite of the h and the k in the equation.
    c. **NOTE**: For an ellipse, the foci are more important than the center.
3. **Direction of major axis** (**Note**: the a is always the larger axis):
    a. If the $x^2$ term is with the $a^2$, then the major axis is horizontal.
    b. If the $y^2$ term is with the $a^2$, then the major axis is horizontal.
4. **Foci**: These are two fixed points inside the ellipse that combine with the lengths of the axes to determine the size of the ellipse.
    a. For horizontal ellipses, the foci are $(h \pm c, k)$.
    b. For vertical ellipses, the foci are $(h, k \pm c)$.
    c. Find "c" by doing $c^2 = a^2 - b^2$. Notice it's different from the Pythagorean Theorem.
5. **Length of the axes**: (**NOTE**: write the length in whatever units are shown, or just "units.")
    a. Major axis: 2a.
    b. Minor axis: 2b.

# 8-5 HYPERBOLAS

- **Hyperbola**: Set of all points in a plane such that the absolute value of the difference of the distances from two fixed points is constant. The fixed points are called foci. (See pictures in the middle of page 441 and at the top of442—especially **442** for all the parts of a hyperbola).

1. **Hyperbola equation**: Standard form: You might have to complete the square in order to put the equation into standard form.

| a. If the transverse axis is horizontal: | b. If the transverse axis is vertical: |
|---|---|
| $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ | $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ |

2. **Center**: (h, k). The central point between the two parts of the hyperbola.
   a. Use the standard form to find h and k.
   b. **HINT**: the sign is opposite of the h and the k in the equation.
   c. **NOTE**: For an hyperbola, the foci and vertices are more important than the center.
3. **Foci**: These are two fixed points, one each inside each part of the hyperbola.
   a. (c, k) and (–c, k) or (h, c) and (h –c).
   b. Find "c" by doing $c^2 = a^2 + b^2$. Notice it's the **same** as the Pythagorean Theorem.
4. **The axes** (**Note**: the a is always the larger axis):
   a. **Transverse axis**
      i. The transverse axis is the length between the two vertices.
      ii. That means the actual distance (length) is calculated as 2a.
      iii. If the $x^2$ term is with the $a^2$, then the transverse axis is horizontal.
      iv. If the $y^2$ term is with the $a^2$, then the transverse axis is vertical.
   b. **Conjugate axis**
      i. Perpendicular to the transverse axis through the center.
      ii. The actual distance (length) of the conjugate axis is calculated as 2b.
5. **Asymptotes**: These are the lines that the hyperbola's branches approach—but never actually touch.

| **The Equations of the Asymptotes:** | |
|---|---|
| a. If the transverse axis is horizontal: | b. If the transverse axis is vertical: |
| $y - k = \pm \frac{b}{a}(x - h)$ | $x - h = \pm \frac{b}{a}(y - k)$ |

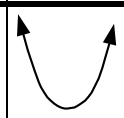
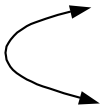
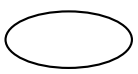
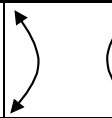
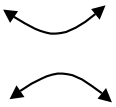
# 8-6 CONIC SECTIONS

1. Standard Form of Conic Sections.
   a. See the chart on page 449.
   b. You normally have to **complete a square** (or even two) to rewrite an equation of a conic section, to change it from the general form into the standard form. Doing that is one way to determine if an equation is a parabola, circle, ellipse, or hyperbola.
2. Identify Conic Sections.
   a. See the chart at the top of page 450. All you need is A and C, and you can determine from the general form (without changing it into standard from) if you have a parabola, circle, ellipse, or hyperbola.

# 8-7 SOLVING QUADRATIC SYSTEMS

1. These are easy—DO NOT make them more complicated!
    a. GRAPHING: Just graph one conic onto a graph, then graph the other conic, then see where they meet.
    b. ALGEBRAICALLY: Solve the two equations, using both equations at the same time, by using the substitution or elimination methods. See section 3-2, where you did these same things with linear equations (straight lines).
2. Inequalities: Just like section 3-3, these must be graphed, not done algebraically.
    a. Solve the two inequalities graphically—one, then the other, just like in the last paragraph.
        i. Pretend temporarily that they are both equations, and solve them like equations in the last paragraph.
    b. Now use the inequality signs to figure out where each conic should be shaded. You should use the point system to figure out where to shade—pick a point and plug it into the inequality to see if it's true or false, and shade the true region.
    c. When you've shaded for both conics, see where they overlap. That's the solution! Make sure you shade that darker than all other regions, so that it's clear what is the solution.

A Helpful Chart to Summarize some of the Above Information:

| *General Quadratic Equation:* $\boldsymbol{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$*, where A, B, and C are not all zero.* | | | | | | | |
|---|---|---|---|---|---|---|---|
| | **Parabola** | **Parabola** | **Circle** | **Ellipse** | **Ellipse** | **Hyperbola** | **Hyperbola** |
| |  |  |  |  |  |  |  |
| Standard Form of Equation | $y = a(x-h) + k$ | $x = a(y-k) + h$ | $(x-h)^2 + (y-k)^2 = 1$ | $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ | $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ | $(x-h)^2/a^2 - (y-k)^2/b^2 = 1$ | $(y-k)^2/a^2 - (x-h)^2/b^2 = 1$ |
| Vertices | (h, k) | (h, k) | Center: (h, k) | (a, h), (–a, h) | (k, b), (k, –b) | (a, h), (–a, h) | (k, a), (k, –a) |
| Foci | (h, k + 1/4a) | (h + 1/4a, k) | | (h+c, k), (h-c, k) | (k, h+c), (k, h-c) | (c, h), (–c, h) | (k, c), (k, –c) |
| Directrix | y = k – (1/4a) | x = y – (1/4a) | | | | | |
| Axis of Symmetry | x = h | y = k | | | | | |
| Direction of opening | Up because a > 0 | Down b/c a < 0 | | | | | |
| Direction of Major Axis | | | | Horizontal | Vertical | | |
| Length of Major Axis | | | | 2a units | 2a units | | |
| Length of Major Axis | | | | 2b units | 2b units | | |
| Direction of Transverse Axis | | | | | | Horizontal | Vertical |
| Length of Transverse Axis | | | | | | 2a units | 2a units |
| Length of Conjugate Axis | | | | | | 2b units | 2b units |
| Equations of Asymptotes | | | | | | $y = \pm(b/a)(x-h)$ | $y = \pm(a/b)(x-h)$ |