

Pre-Algebra, Algebra

GRAPHING LINEAR EQUATIONS

1. **Linear equation.** $Ax + By = C$. This is called the “standard form” of a linear equation. Linear simply means line. Finding and using the equation of a line is this topic.
2. **Slope-intercept equation:** $y = mx + b$. This is another form of a linear equation. This form is the most useful form; it allows you to graph a line in the easiest way.
3. **Y-intercept:**
 - a. b. In the equation $y = 3x - 4$, the y-intercept is -4 .
 - b. It is always the **constant** in the slope-intercept equation.
 - c. Use it to put a point on the graph:
 - i. This number is the place where the line (the line you’re trying to graph) intercepts (or “cuts through”) the y-axis. That’s how it gets its name: “y-intercept,” short for “intercept of the y-axis.”
 - ii. Realize also that the y-intercept is the y value when the x value is zero. The x value is always zero for points that are anywhere on the y-axis. In the equation, when x is zero, it multiplies to the slope, but multiplying by zero makes zero, so the only thing left in the equation is $y = b$.
 - iii. **So just go to that point on the y-axis, and put a dot there.**
4. **Slope:**
 - a. m.
 - b. Always the coefficient of the x term in the slope-intercept equation.
 - c. Always a fraction: Rise over run. If not a fraction, put 1 underneath the number.
 - d. Rise: up if positive, down if negative. Follow the y-axis numbers!
 - e. Run: right if positive, left if negative. Follow the x-axis numbers!
 - f. **USE THE SLOPE (AND Y-INTERCEPT) TO GRAPH A LINE:**
 - i. Start at any point you’re given that’s on a line. Most of the time, you’re working with $y = mx + b$, so **use the b** (see above for graphing the y-intercept).
 - ii. From that point (the y-intercept), use the “**rise**” number to go up or down.
 - iii. When you get as far as the rise allows you, then go right or left, as far as the “**run**” number tells you to go.
 - iv. Put a dot there, at that point.
 - v. You now have two points. It helps to make another point, or even two more, just so it’s easier to draw your line.
 - vi. Draw a line through the points, through the whole graph.
 - vii. Don’t forget arrows.
5. **Solve for y.** You are not always getting equations in the form $y = mx + b$. You have to put them into that form. You do that by solving for y. This means moving things around just like you’ve done with equations in the past. Example:
 - a. $-6x - 2y = 8$. Use $+6x$ on both sides. Do NOT use $--6$. It’s confusing, and $--$ means $+$, so use $+$.
 - b. $-6x + 6x - 2y = +6x + 8$. That becomes $-2y = 6x + 8$. ALWAYS put the x term right after the equal sign, because that gets you closer to the $y = mx + b$ form.
 - c. You want to get y by itself, so now you have to divide both sides by -2 (coefficient of y).
 - d. $\frac{-2y}{-2} = \frac{6x}{-2} + \frac{8}{-2}$ Notice that you divide ALL terms by the coefficient.
 - e. Now you have $y = -3x - 4$. You can identify the y-intercept (-4) and the slope (-3) .

6. **Slope formula.** When you are given two points, use the slope formula: $\frac{y_2 - y_1}{x_2 - x_1}$
- The “sub” numbers refer to the separate points. Choose one point to be the first point, and put x_1 and y_1 above the numbers, then put x_2 and y_2 above the second pair of numbers.
 - Now just simply plug the numbers in.
 - BE CAREFUL OF NEGATIVES.
 - You will often get something like $-3 - -1$.
 - That $- -$ means +; ALWAYS change it immediately to plus.
 - So $-3 - -1$ is $-3 + 1$, which is -2 .
 - WARNING: Remember to put the y numbers on top!
 - Result: You will always get a fraction. If the denominator is 1, you don't have to show the slope as a fraction, but in reality, it IS a fraction, because the slope shows the rise over the run.
 - Special cases:
 - You get a zero in the numerator (top). That simply means the slope = 0, because 0 divided by (“over”) anything is 0. This means that the line is horizontal (\leftrightarrow).
 - You get a zero in the denominator (bottom). This means the fraction is undefined, because you can never have zero as the denominator (can not divide by zero). This means there is no slope, which means the line is vertical (\updownarrow).

7. **Identify slope and y-intercept in an equation.**

- You must put the equation into slope-intercept form ($y = mx + b$) first.
- The b is the y-intercept. Make sure the sign of it stays with it (a minus sign actually means it's a negative number. Example: -4 is really $+^{-}4$. However, it's much easier and far more useful to write $+^{-}4$ as -4).
- The slope is the coefficient of the x.
 - If it's negative, keep the negative sign with it. DO NOT include the x. The x is not the slope. Only the coefficient is the slope.
 - If there's a fraction with the x, then the whole fraction is the slope.

8. **Write a rule. Rule means equation. So, write an equation.** The equation will be $y = mx + b$.

- Find m (slope). There are 2 ways:
 - Formula. Use two points and the slope formula: $\frac{y_2 - y_1}{x_2 - x_1}$
 - Visually. If you have a graphed line, look at the rise, then the run. You do this by picking one definite point, then go up (rise) and to the right or left (run). Count carefully.
- Find b (y-intercept). There are 2 ways:
 - Use the slope intercept equation.
 - Fill in m.
 - Put in a point (an x and a y, that you're given or can see on a graph).
 - Solve for b.
 - Visually. If you have a graphed line, look at where the line intercepts the y-axis. That number is the b (the “y-intercept”).
- Example:** *Given the two points $(-5, 2)$ and $(1, -4)$, write the equation of the line.*
 - Use the **slope formula**. The result shows you that the slope is -1 .
 - Put the slope into the **equation form**: $y = -1x + b$.
 - Use either point, and **put the point into the equation also, as x and y**: $2 = -1(-5) + b$.
 - Solve for b**. $2 = 5 + b$, which means $b = -3$.
 - Write $y = mx + b$ with the correct m and b**: $y = -x - 3$. That's the “rule” or equation.