

# **ASTEM97**

**Based on the  
IAPWS IF-97**

## **Water and Steam Properties for Industrial Use**

Implementation by

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Appendix E

## **Backward Equations**



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## Introduction

In the development of backward equations for pressure and temperature based on input pairs of  $(h,s)$ ,  $(v,h)$  and  $(v,s)$ , the boundaries between IF-97 regions present some difficulty in determining the state point location. In ASTEM97 the boundary between region 1 and region 3 on the 623.15 K line is set by the Gibb's equation for region 1. Likewise for the boundary between region 2 and region 3 on the "2-3 boundary" line is set by the Gibb's equation for region 2.

In general, with the exception of the very low pressure region from 611 Pa to about 1,080 Pa, these backward equations predict the pressure to within 0.1 % near region boundaries and significantly better elsewhere.

Figures 1 through 3 compare the differences in the enthalpy, entropy and specific volume on these boundaries as a function of pressure. The thick line, T23B, shows the boundary between regions 2 and 3 and the thin line, T623, shows the boundary between regions 1 and region 3. The region 3 properties are obtained from the Helmholtz equation at the same pressure and temperature used to evaluate the Gibb's equations.

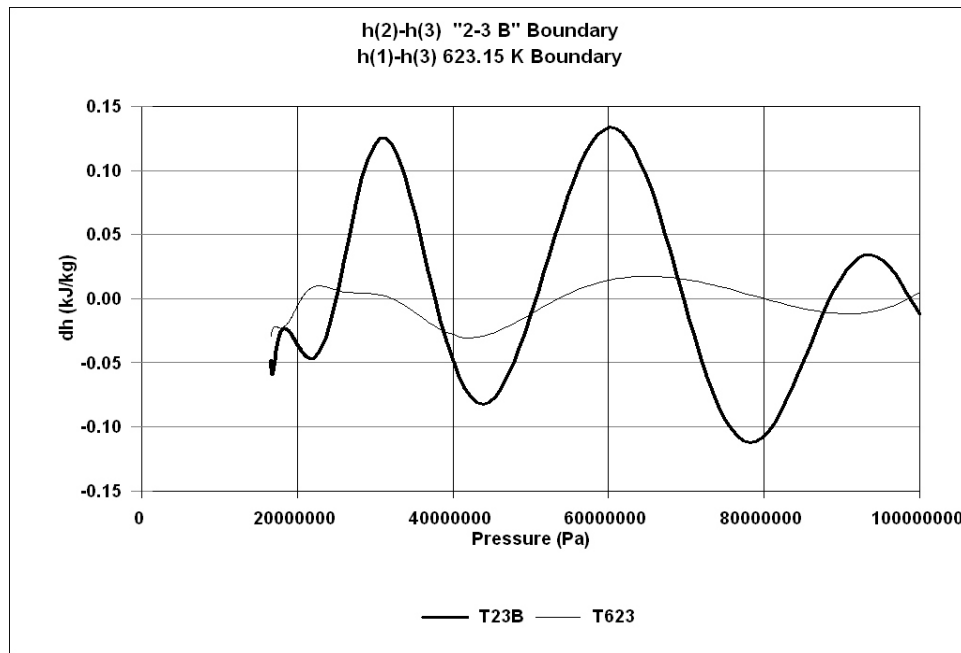


Figure 1 - Enthalpy difference on T623 and T23B boundaries

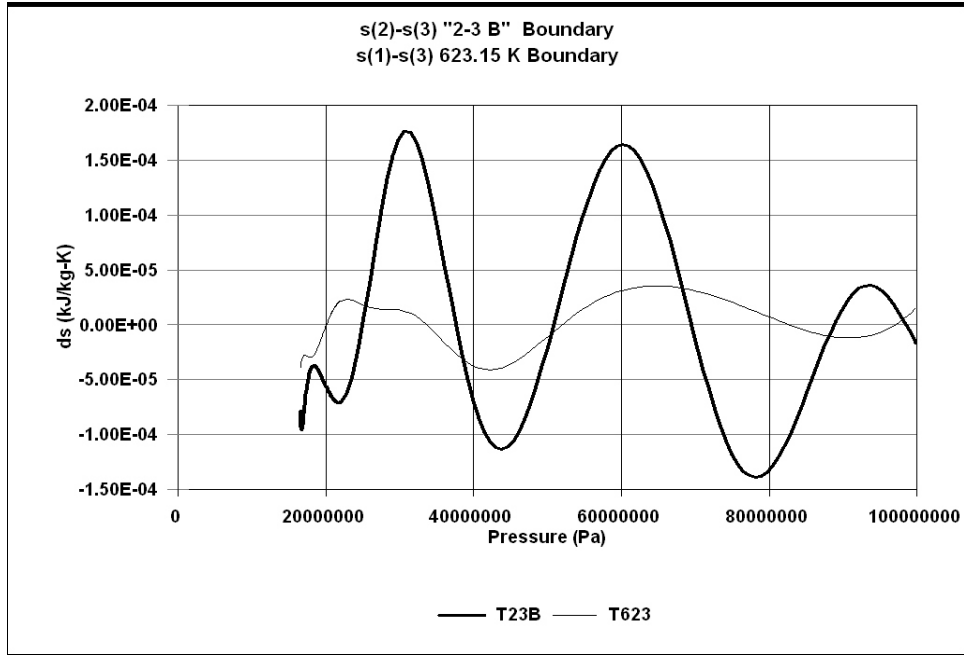


Figure 2 - Entropy difference on T623 and T23B boundaries

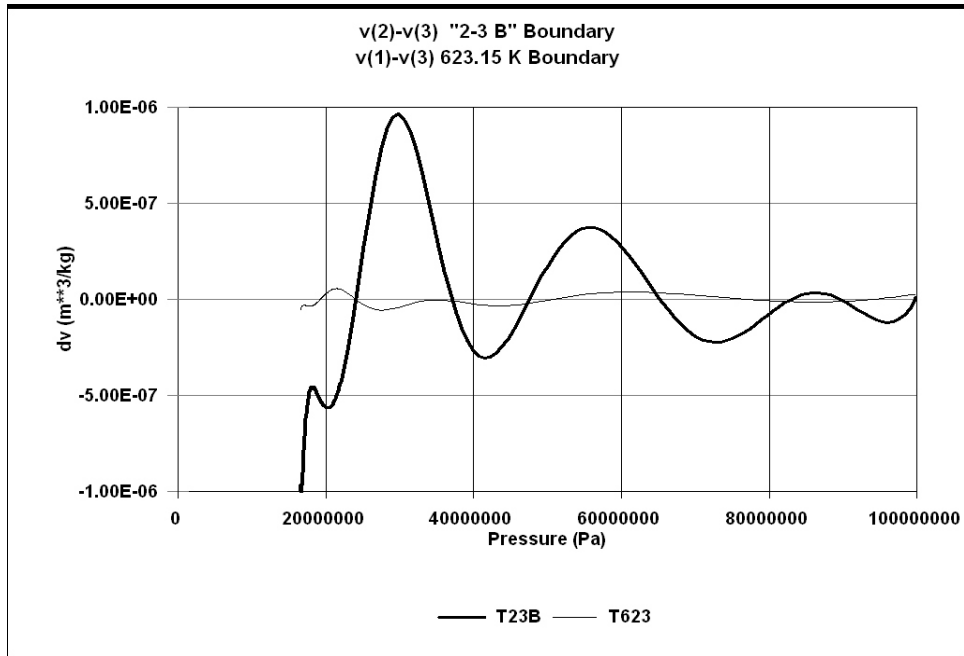


Figure 3 - Specific volume difference on T23B and T623 boundaries

The estimated error in the pressure predictions for the backward equations developed for use in **ASTEM97** are based on 1,000 equally spaced points within selected pressure ranges. The estimated error range is -0.0645 to + 0.0680 per cent, near the T23B and T623 boundaries and near the saturated fluid line at very low pressures. See Tables 1 through 6.

## Backward Equations for p(h,s) and T(h,s)

p(h,s) first determines the possible pressure range based on the input value for the enthalpy, h, in kJ/kg. The pressure range is then searched to determine the pressure at which the input value for the entropy, s, in kJ/kg-K, is found. On the boundary between region 2 and region 3, the entropy presents some difficulties, as shown in Figure 4.

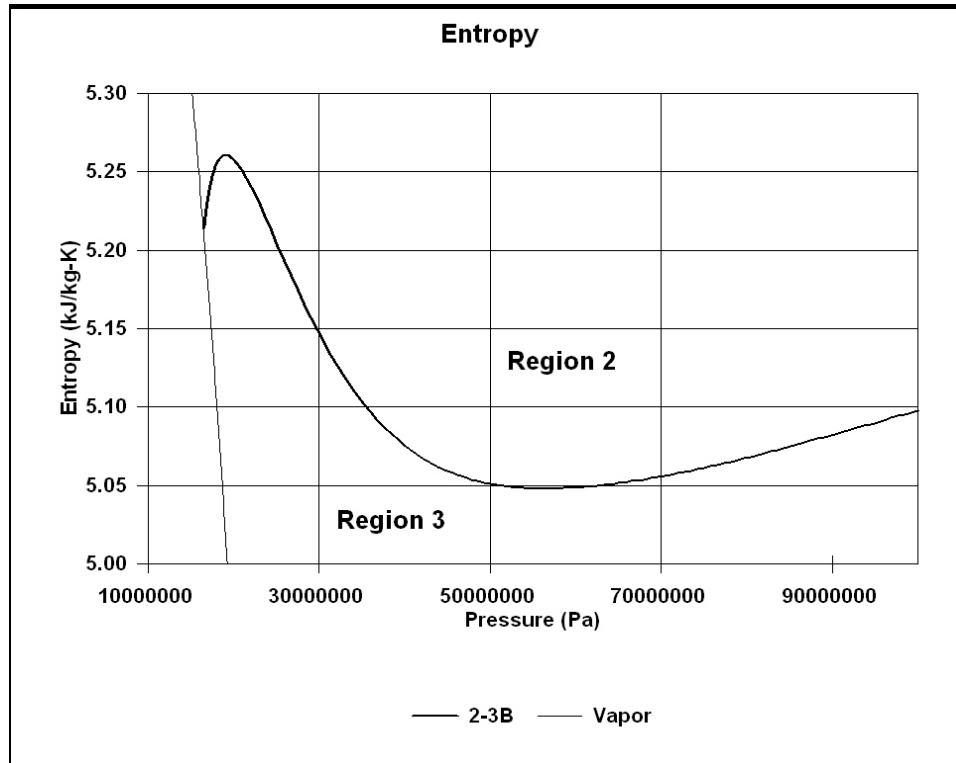


Figure 4 - Entropy on the 2-3 boundary

While p(h,s) does well on the boundary (E23b), near the boundary the error increases, as shown in Table 1. On the 623.15 K (E623) boundary, the behavior is similar.  $\Delta T_2$  is  $1.0 \times 10^{-7}$  K.

Table 1 - p(h,s) pressure error on 623.15 K and 23B boundaries in percent

Pressure	T (K)	623.15- $\Delta T_2$	623.15	623.15+ $\Delta T_2$	T23B- $\Delta T_2$	T23B	T23B+ $\Delta T_2$
Range							
16.53 MPa	Min	-8.49e-07	-4.87e-03	-1.01e-02	1.65e-07	-6.99e-04	-4.87e-03
to	Max	4.94e-07	5.08e-07	1.41e-02	6.99e-04	3.63e-06	1.02e-07
22.064 MPa	Average	-7.54e-08	-4.75e-06	3.43e-03	3.84e-04	-2.61e-04	-2.65e-04
	Stand Dev	2.72e-07	1.54e-04	7.74e-03	1.91e-04	2.87e-04	3.22e-04
22.064 MPa	Min	-5.85e-06	-1.02e-06	-1.09e-02	-5.95e-02	-6.45e-02	-6.45e-02
to	Max	3.94e-07	1.03e-06	2.10e-02	2.13e-03	3.40e-05	5.40e-05
100 MPa	Average	-4.13e-07	1.77e-08	7.88e-04	-5.31e-03	-8.60e-03	-8.60e-03
	Stand Dev	9.25e-07	2.21e-07	8.28e-03	1.60e-02	1.82e-02	1.82e-02

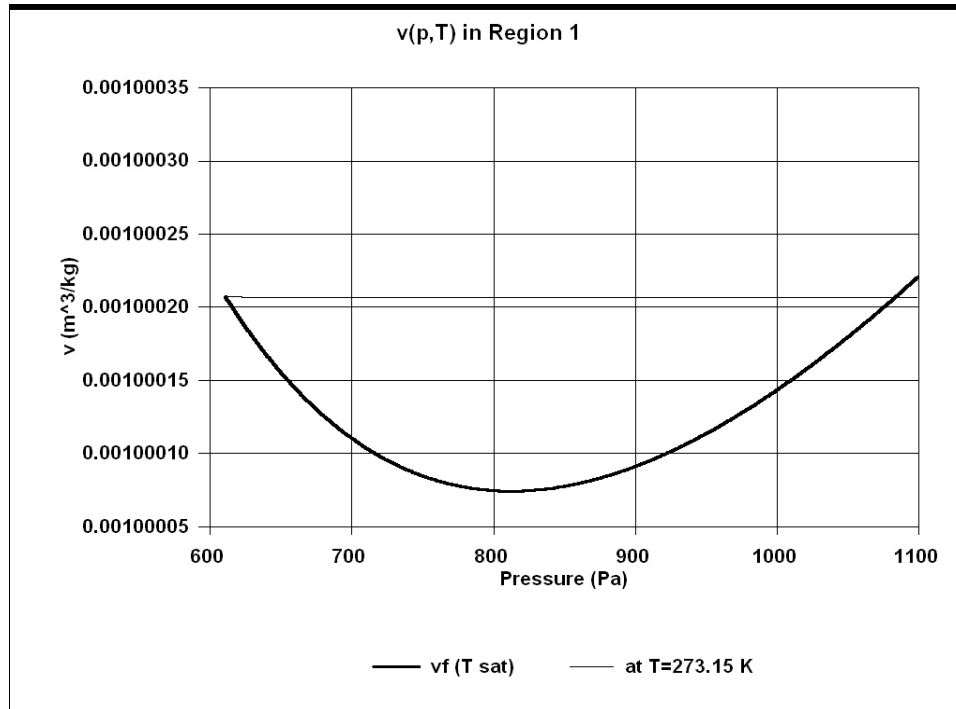
Table 2 shows the pressure error, in percent, for the other regions, including the 2-phase region at a quality (X) of 0.5.  $\Delta T1$  is 0.01 K.

**Table 2 - p(h,s) pressure error in percent**

Pressure	T (K)	273.15	1073.15	2273.15	Tf	Tf- $\Delta T1$	Tg	Tg+ $\Delta T1$	X=0.5
Range									
611.12 Pa	Min	-1.08e-02	-2.88e-09	-2.17e-06	-4.46e-08	-2.82e-03	-2.83e-08	-7.23e-08	-3.68e-04
to	Max	1.22e-02	3.16e-09	2.15e-06	4.48e-08	6.80e-02	2.82e-08	3.07e-08	3.84e-04
10000 Pa	Average	8.62e-04	9.00e-12	-2.22e-08	7.46e-10	5.37e-02	5.81e-10	-8.74e-11	-1.83e-06
	Stand Dev	2.28e-03	4.82e-10	1.07e-06	2.25e-08	1.30e-02	1.40e-08	4.96e-09	1.83e-04
10000 Pa	Min	-2.76e-04	-1.11e-10	-2.16e-06	-3.31e-08	-6.33e-06	-2.99e-08	-9.65e-09	-2.95e-04
to	Max	5.18e-04	1.81e-08	2.16e-06	3.34e-08	5.11e-02	2.99e-08	9.74e-09	2.81e-04
1 MPa	Average	1.35e-05	7.10e-09	5.51e-08	-4.54e-10	5.33e-03	-6.89e-10	1.17e-10	-1.22e-06
	Stand Dev	5.22e-05	3.85e-09	1.17e-06	1.37e-08	1.34e-02	1.52e-08	3.85e-09	1.22e-04
1 MPa	Min	-4.87e-06	1.12e-11	-2.14e-06	-2.25e-08	-6.91e-07	-2.81e-08	-1.29e-08	-1.88e-04
to	Max	8.95e-06	1.17e-08	2.16e-06	2.24e-08	1.02e-06	2.82e-08	1.33e-08	1.95e-04
10 MPa	Average	7.62e-07	4.22e-09	3.60e-08	-5.46e-10	-2.13e-09	-2.20e-10	-4.54e-12	-2.99e-07
	Stand Dev	1.77e-06	2.37e-09	1.13e-06	9.77e-09	1.63e-07	1.15e-08	4.94e-09	8.50e-05
10 MPa	Min	-5.62e-07	-2.01e-08		-1.37e-08	-2.34e-07	-1.56e-08	-2.13e-08	-1.32e-04
to	Max	1.33e-06	1.98e-08		1.38e-08	2.79e-07	1.49e-08	2.48e-08	1.33e-04
16.53 MPa	Average	2.86e-07	6.11e-10		-9.75e-11	2.71e-09	7.90e-11	6.56e-11	-1.00e-06
	Stand Dev	4.29e-07	9.53e-09		6.16e-09	8.38e-08	6.69e-09	7.15e-09	5.93e-05
16.53 MPa	Min	-4.66e-07	-1.91e-08		-1.63e-03	-2.64e-06	-4.87e-03	-2.66e-06	-9.76e-05
to	Max	7.12e-07	1.89e-08		9.34e-09	2.43e-06	2.05e-07	4.01e-06	9.75e-05
22.064 MPa	Average	1.13e-07	-1.83e-10		-4.67e-06	-6.02e-09	-5.01e-06	4.05e-09	-7.67e-07
	Stand Dev	2.84e-07	9.43e-09		8.52e-05	2.11e-07	1.54e-04	2.40e-07	4.19e-05
22.064 MPa	Min	-3.08e-07	-1.80e-08						
to	Max	5.42e-07	1.71e-08						
100 MPa	Average	4.18e-08	9.19e-11						
	Stand Dev	1.18e-07	8.35e-09						

### Backward equations for $p(v,h)$ , $T(v,h)$ , $p(v,s)$ and $T(v,s)$

As shown in Figure 5, the saturated fluid specific density presents difficulty in developing the backward equations based on  $(v,h)$  and  $(v,s)$  for the very low pressure range of 611 Pa to about 1,080 Pa.



**Figure 5 - Specific volume in region 1 at low pressures**

Tables 3 thru 6 show the pressure error in percent for the  $p(v,h)$  and  $p(v,s)$  backward equations.  $\Delta T2$  in Tables 3 and 4 is  $1.0 \times 10^{-7}$  K.  $\Delta T1$  in Tables 5 and 6 is 0.01 K.

**Table 3 -  $p(v,h)$  pressure error on 623.15 K and 23B boundaries in percent**

Pressure	T (K)	623.15- $\Delta T2$	623.15	623.15+ $\Delta T2$	T23B- $\Delta T2$	T23B	T23B+ $\Delta T2$
Range							
16.53 MPa	Min	-4.05e-08	-3.24e-09	-1.21e-07	1.83e-07	-4.31e-03	-1.29e-08
	to	1.83e-07	3.18e-09	7.99e-08	4.88e-03	1.60e-08	6.30e-09
22.064 MPa	Average	8.11e-10	2.83e-11	2.54e-11	2.15e-03	-8.52e-06	-5.52e-10
	Stand Dev	1.51e-08	1.51e-09	2.30e-08	8.16e-04	1.90e-04	5.08e-09
22.064 MPa	Min	-3.89e-02	-5.40e-09	-6.80e-08	-9.19e-03	-1.05e-09	-9.97e-09
	to	1.76e-02	5.44e-09	5.70e-08	1.06e-02	1.02e-09	9.56e-09
100 MPa	Average	4.96e-05	-7.03e-12	-2.19e-11	-5.07e-04	-9.81e-13	8.22e-11
	Stand Dev	1.15e-02	2.70e-09	8.20e-09	4.55e-03	3.59e-10	3.55e-09

**Table 4 - p(v,s) pressure error on 623.15 K and 23B boundaries in percent**

Pressure	T (K)	623.15-ΔT2	623.15	623.15+ΔT2	T23B-ΔT2	T23B	T23B+ΔT2
Range							
16.53 MPa	Min	-1.16e-07	-3.24e-09	-1.56e-07	1.83e-07	-1.73e-09	-2.31e-08
to	Max	1.83e-07	3.18e-09	1.36e-07	5.91e-03	2.52e-08	1.29e-07
22.064 MPa	Average	2.53e-09	2.83e-11	-9.68e-09	2.61e-03	1.89e-10	-4.18e-09
	Stand Dev	4.21e-08	1.51e-09	4.36e-08	9.93e-04	1.87e-09	7.27e-09
22.064 MPa	Min	-5.98e-02	-5.40e-09	-9.65e-08	-1.30e-02	-1.05e-09	-2.31e-08
to	Max	1.69e-02	5.44e-09	9.94e-08	1.26e-02	1.02e-09	1.93e-08
100 MPa	Average	-5.37e-03	-7.03e-12	2.02e-09	-1.80e-03	-9.81e-13	3.53e-11
	Stand Dev	1.46e-02	2.70e-09	2.61e-08	5.35e-03	3.59e-10	9.29e-09

**Table 5 - p(v,h) pressure error in percent**

Pressure	T (K)	273.15	1073.15	2273.15	Tf	Tf-ΔT1	Tg	Tg-ΔT1	X=0.5
Range									
611.12 Pa	Min	-2.82e-03	-2.48e-08	-2.30e-08	-2.82e-03	-2.82e-03	-2.30e-08	-7.23e-08	-2.01e-06
to	Max	2.53e-03	2.02e-08	2.39e-08	4.63e-07	2.53e-03	2.39e-08	3.90e-08	2.03e-06
10000 Pa	Average	-5.31e-06	-7.57e-10	-2.30e-10	-6.20e-05	5.84e-06	-2.30e-10	-5.51e-11	1.89e-08
	Stand Dev	4.26e-04	4.76e-09	5.22e-09	3.13e-04	3.64e-04	5.22e-09	5.26e-09	1.02e-06
10000 Pa	Min	-9.11e-05	-8.79e-10	-8.79e-10	-8.49e-08	-4.16e-05	-8.79e-10	-3.38e-09	-1.98e-06
to	Max	1.33e-04	9.20e-10	9.20e-10	1.02e-07	1.20e-04	9.20e-10	3.76e-09	1.98e-06
1 MPa	Average	8.62e-08	1.60e-12	1.29e-12	3.98e-11	2.77e-07	1.08e-12	8.36e-11	-2.08e-08
	Stand Dev	9.81e-06	9.08e-11	9.09e-11	1.70e-08	6.08e-06	9.06e-11	1.27e-09	9.65e-07
1 MPa	Min	-1.64e-06	-2.06e-11	-2.06e-11	-1.46e-08	-6.40e-07	-4.03e-11	-5.90e-09	-1.79e-06
to	Max	1.76e-06	1.98e-11	1.98e-11	1.58e-08	8.69e-07	4.34e-11	6.24e-09	1.76e-06
10 MPa	Average	8.37e-10	6.20e-14	-9.38e-14	4.39e-11	1.66e-09	1.44e-13	4.49e-11	-3.72e-08
	Stand Dev	3.38e-07	7.12e-12	4.43e-12	3.95e-09	9.53e-08	1.51e-11	1.85e-09	8.25e-07
10 MPa	Min	-1.92e-07	-3.20e-11		-2.69e-09	-1.11e-07	-6.62e-11	-1.06e-08	-1.28e-06
to	Max	1.92e-07	3.16e-11		2.50e-09	1.06e-07	6.43e-11	1.43e-08	1.23e-06
16.53 MPa	Average	-2.24e-10	8.24e-14		-3.39e-11	3.87e-10	-1.07e-13	-1.55e-10	-3.85e-09
	Stand Dev	8.62e-08	1.46e-11		1.03e-09	3.50e-08	2.97e-11	3.76e-09	6.00e-07
16.53 MPa	Min	-1.22e-07	-4.33e-11		-1.85e-03	-2.64e-06	-5.06e-05	-2.66e-06	-2.38e-03
to	Max	1.22e-07	4.43e-11		9.61e-10	2.43e-06	3.02e-06	4.01e-06	1.06e-05
22.064 MPa	Average	-3.22e-10	2.70e-14		-5.66e-06	-6.14e-09	-6.99e-08	1.87e-09	-2.39e-06
	Stand Dev	6.03e-08	2.05e-11		9.40e-05	1.95e-07	1.75e-06	2.27e-07	7.51e-05
22.064 MPa	Min	-9.14e-08	-2.14e-10						
to	Max	8.70e-08	2.06e-10						
100 MPa	Average	7.78e-11	-3.97e-13						
	Stand Dev	2.68e-08	7.32e-11						

**Table 6 - p(v,s) pressure error in percent**

Pressure	T (K)	273.15	1073.15	2273.15	Tf	Tf-ΔT1	Tg	Tg-ΔT1	X=0.5
Range									
611.12 Pa	Min	-2.82e-03	-2.30e-08	-2.30e-08	-2.82e-03	-2.82e-03	-2.30e-08	-7.23e-08	-2.14e-06
to	Max	2.53e-03	2.39e-08	2.39e-08	1.37e-04	6.80e-02	2.39e-08	3.07e-08	2.15e-06
10000 Pa	Average	-5.31e-06	-2.30e-10	-2.30e-10	-5.82e-05	5.37e-02	-2.30e-10	-8.74e-11	1.51e-08
	Stand Dev	4.26e-04	5.22e-09	5.22e-09	3.09e-04	1.30e-02	5.22e-09	4.96e-09	1.09e-06
10000 Pa	Min	-9.11e-05	-8.79e-10	-8.79e-10	-8.49e-08	-6.33e-06	-8.79e-10	-9.65e-09	-2.11e-06
to	Max	1.33e-04	9.20e-10	9.20e-10	1.02e-07	5.11e-02	9.20e-10	9.74e-09	2.12e-06
1 MPa	Average	8.62e-08	1.60e-12	1.29e-12	3.98e-11	5.33e-03	1.08e-12	1.17e-10	-2.39e-08
	Stand Dev	9.81e-06	9.08e-11	9.09e-11	1.70e-08	1.34e-02	9.06e-11	3.85e-09	1.05e-06
1 MPa	Min	-1.64e-06	-2.06e-11	-2.06e-11	-1.46e-08	-6.91e-07	-4.03e-11	-1.29e-08	-1.96e-06
to	Max	1.76e-06	1.98e-11	1.98e-11	1.58e-08	1.02e-06	4.34e-11	1.33e-08	2.00e-06
10 MPa	Average	8.37e-10	6.20e-14	-9.38e-14	4.39e-11	-2.13e-09	1.44e-13	-4.54e-12	1.63e-08
	Stand Dev	3.38e-07	7.12e-12	4.43e-12	3.95e-09	1.63e-07	1.51e-11	4.94e-09	8.97e-07
10 MPa	Min	-1.92e-07	-3.20e-11		-2.69e-09	-2.34e-07	-6.62e-11	-2.13e-08	-1.34e-06
to	Max	1.92e-07	3.16e-11		2.50e-09	2.79e-07	6.43e-11	2.48e-08	1.33e-06
16.53 MPa	Average	-2.24e-10	8.24e-14		-3.39e-11	2.71e-09	-1.07e-13	6.56e-11	1.24e-08
	Stand Dev	8.62e-08	1.46e-11		1.03e-09	8.38e-08	2.97e-11	7.15e-09	6.25e-07
16.53 MPa	Min	-1.22e-07	-4.33e-11		-1.95e-03	-2.64e-06	-5.06e-05	-2.66e-06	-2.82e-03
to	Max	1.22e-07	4.43e-11		2.39e-05	2.43e-06	3.02e-06	4.01e-06	1.06e-05
22.064 MPa	Average	-3.22e-10	2.70e-14		-1.93e-06	-6.02e-09	-6.99e-08	4.05e-09	-2.84e-06
	Stand Dev	6.03e-08	2.05e-11		6.15e-05	2.11e-07	1.75e-06	2.40e-07	8.93e-05
22.064 MPa	Min	-9.14e-08	-2.14e-10						
to	Max	8.70e-08	2.06e-10						
100 MPa	Average	7.78e-11	-3.97e-13						
	Stand Dev	2.68e-08	7.32e-11						