

ASTEM97

**Based on the
IAPWS IF-97**

Water and Steam Properties for Industrial Use

Implementation by

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Appendix E

Backward Equations

Version 2.0

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Introduction

In the development of backward equations for pressure and temperature based on input pairs of (h,s), (v,h) and (v,s), the boundaries between IF-97 regions present some difficulty in determining the state point location. In **ASTEM97** the boundary between Region 1 and Region 3 on the 623.15 K line is set by the Gibb's equation for Region 1. Likewise for the boundary between Region 2 and Region 3 on the "2-3 boundary" line is set by the Gibb's equation for Region 2.

In general, with the exception of the very low pressure region from 611.213 Pa to about 1,080 Pa, these backward equations predict the pressure to within 0.1 % near region boundaries and significantly better elsewhere.

Figures 1 through 3 compare the differences in the enthalpy, entropy and specific volume on these boundaries as a function of pressure. The thick line, T23B, shows the boundary between Regions 2 and 3 and the thin line, T623, shows the boundary between Region 1 and Region 3. The Region 3 properties are obtained from the Helmholtz equation at the same pressure and temperature used to evaluate the Gibb's equations.

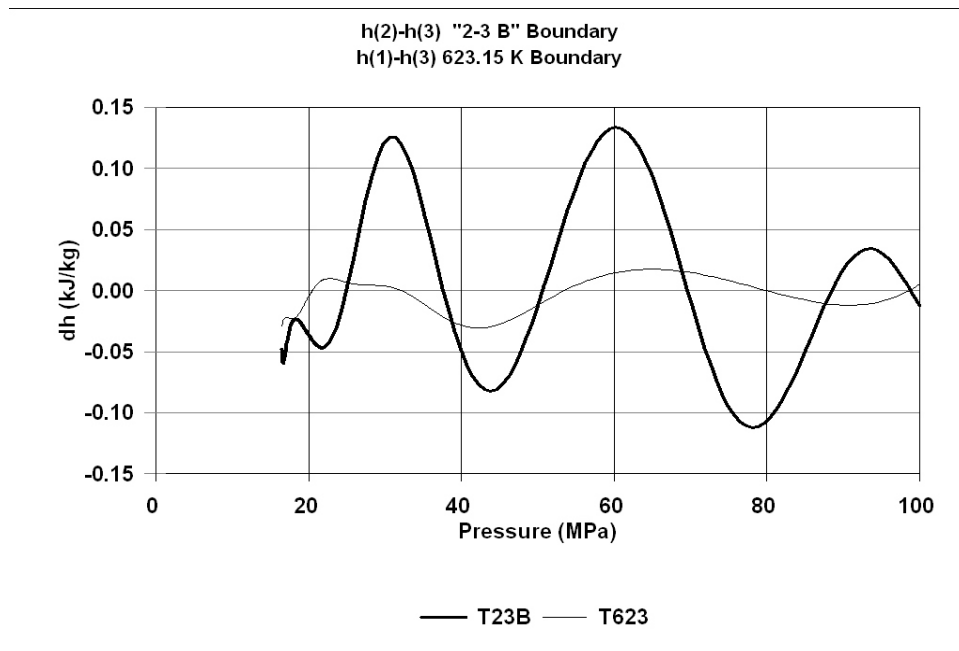


Figure 1 - Enthalpy difference on T623 and T23B boundaries

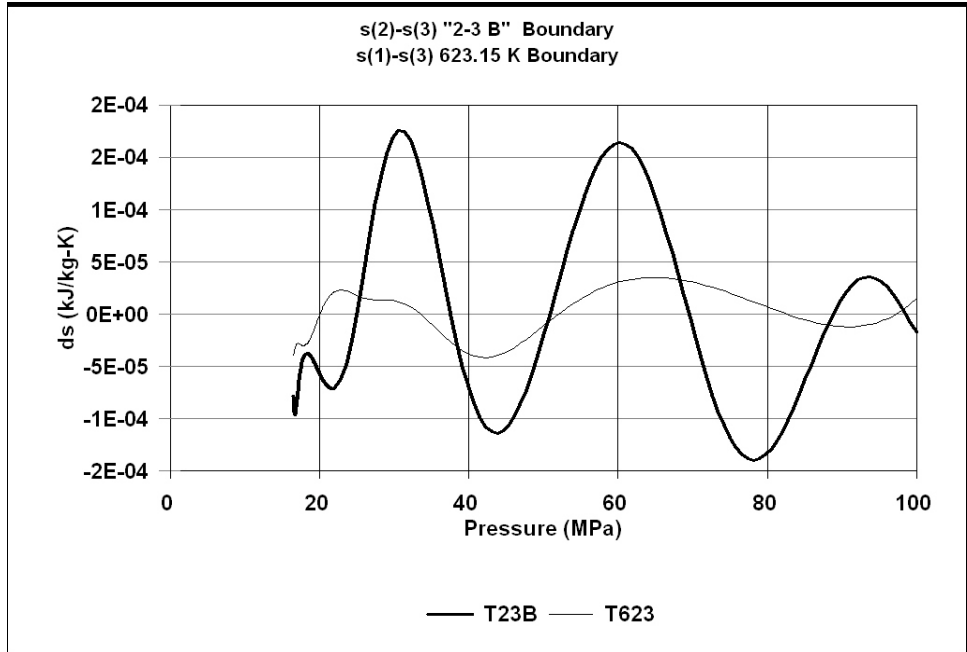


Figure 2 - Entropy difference on T623 and T23B boundaries

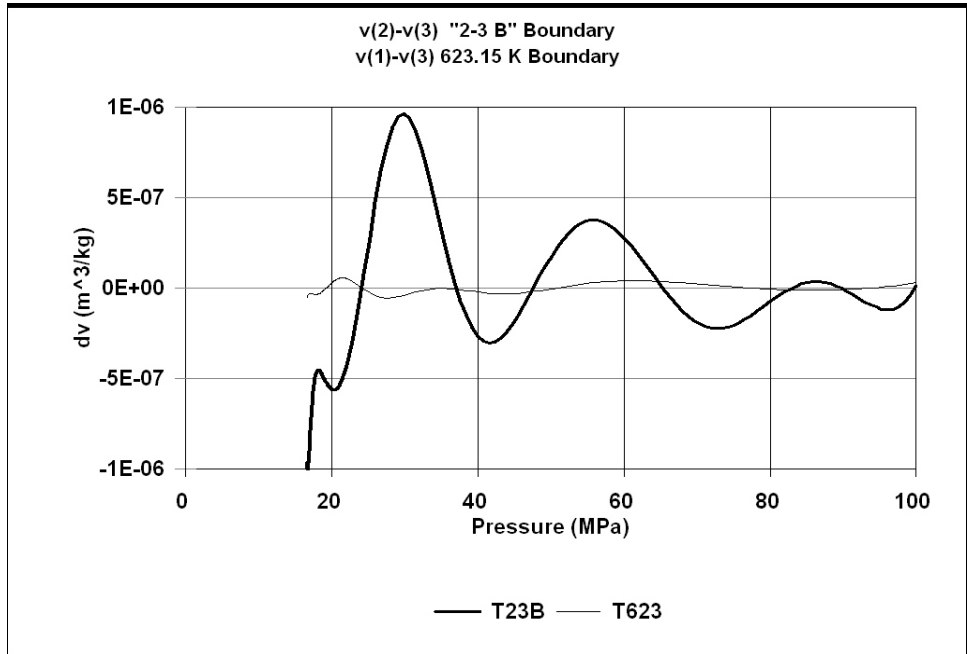


Figure 3 - Specific volume difference on T23B and T623 boundaries

The estimated error in the pressure predictions for the backward equations developed for use in **ASTEM97** are based on 1,000 equally spaced points within selected pressure ranges. The estimated error range is -0.0645 to +0.0680 per cent, near the T23B and T623 boundaries and near the saturated fluid line at very low pressures. See Tables 1 through 6.

ASTEM97 Backward Equations for p(h,s) and T(h,s)

p(h,s) first determines the possible pressure range based on the input value for the enthalpy, h , in kJ/kg. The pressure range is then searched to determine the pressure at which the input value for the entropy, s , in kJ/kg-K, is found. On the boundary between Region 2 and Region 3, the entropy presents some difficulties, as shown in Figure 4.

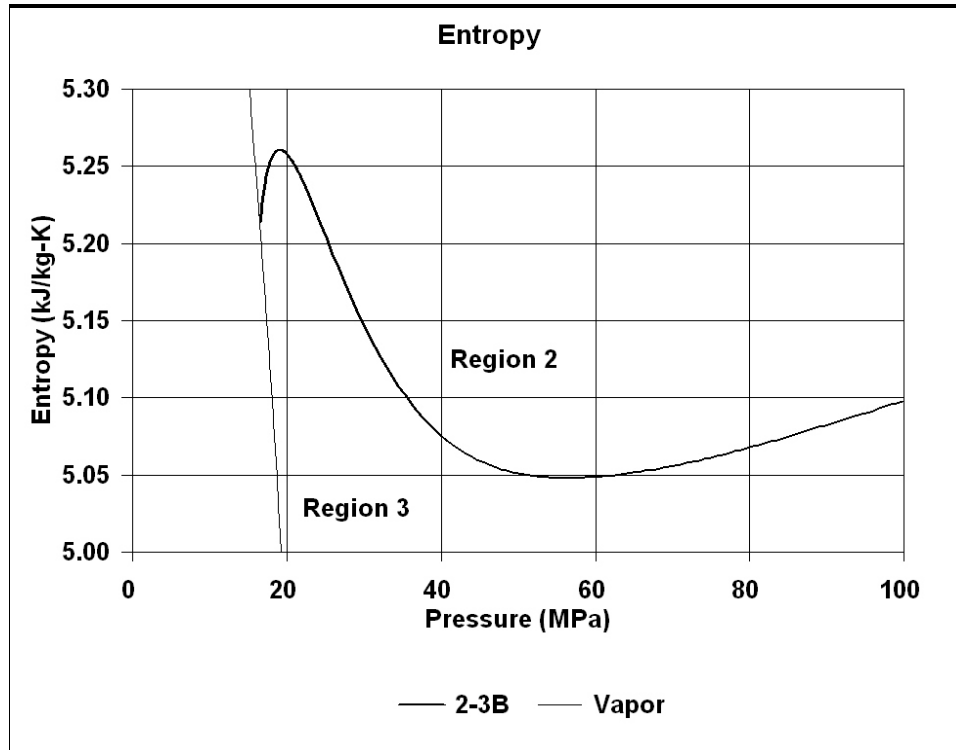


Figure 4 - Entropy on the 2-3 boundary

While p(h,s) does well on the boundary (T23b), near the boundary the error increases, as shown in Table 1. On the 623.15 K (T623) boundary, the behavior is similar. $\Delta T2$ is 1.0×10^{-7} K.

Table 1 - p(h,s) pressure error on 623.15 K and T23B boundaries in percent

Pressure	T (K)	623.15- $\Delta T2$	623.15	623.15+ $\Delta T2$	T23B- $\Delta T2$	T23B	T23B+ $\Delta T2$
Range							
16.53 MPa	Min	-8.49e-07	-4.87e-03	-1.01e-02	1.65e-07	-6.99e-04	-4.87e-03
to	Max	4.94e-07	5.08e-07	1.41e-02	6.99e-04	3.63e-06	1.02e-07
22.064 MPa	Average	-7.54e-08	-4.75e-06	3.43e-03	3.84e-04	-2.61e-04	-2.65e-04
	Stand Dev	2.72e-07	1.54e-04	7.74e-03	1.91e-04	2.87e-04	3.22e-04
22.064 MPa	Min	-5.85e-06	-1.02e-06	-1.09e-02	-5.95e-02	-6.45e-02	-6.45e-02
to	Max	3.94e-07	1.03e-06	2.10e-02	2.13e-03	3.40e-05	5.40e-05
100 MPa	Average	-4.13e-07	1.77e-08	7.88e-04	-5.31e-03	-8.60e-03	-8.60e-03
	Stand Dev	9.25e-07	2.21e-07	8.28e-03	1.60e-02	1.82e-02	1.82e-02

Table 2 shows the pressure error, in percent, for the other regions, including the 2-phase region at a quality (X) of 0.5. $\Delta T1$ is 0.01 K.

Table 2 - p(h,s) pressure error in percent

Pressure	T (K)	273.15	1073.15	2273.15	Tf	Tf- $\Delta T1$	Tg	Tg+ $\Delta T1$	X=0.5
Range									
611.213 Pa	Min	-1.08e-02	-2.88e-09	-2.17e-06	-4.46e-08	-2.82e-03	-2.83e-08	-7.23e-08	-3.68e-04
to	Max	1.22e-02	3.16e-09	2.15e-06	4.48e-08	6.80e-02	2.82e-08	3.07e-08	3.84e-04
10000 Pa	Average	8.62e-04	9.00e-12	-2.22e-08	7.46e-10	5.37e-02	5.81e-10	-8.74e-11	-1.83e-06
	Stand Dev	2.28e-03	4.82e-10	1.07e-06	2.25e-08	1.30e-02	1.40e-08	4.96e-09	1.83e-04
10000 Pa	Min	-2.76e-04	-1.11e-10	-2.16e-06	-3.31e-08	-6.33e-06	-2.99e-08	-9.65e-09	-2.95e-04
to	Max	5.18e-04	1.81e-08	2.16e-06	3.34e-08	5.11e-02	2.99e-08	9.74e-09	2.81e-04
1 MPa	Average	1.35e-05	7.10e-09	5.51e-08	-4.54e-10	5.33e-03	-6.89e-10	1.17e-10	-1.22e-06
	Stand Dev	5.22e-05	3.85e-09	1.17e-06	1.37e-08	1.34e-02	1.52e-08	3.85e-09	1.22e-04
1 MPa	Min	-4.87e-06	1.12e-11	-2.14e-06	-2.25e-08	-6.91e-07	-2.81e-08	-1.29e-08	-1.88e-04
to	Max	8.95e-06	1.17e-08	2.16e-06	2.24e-08	1.02e-06	2.82e-08	1.33e-08	1.95e-04
10 MPa	Average	7.62e-07	4.22e-09	3.60e-08	-5.46e-10	-2.13e-09	-2.20e-10	-4.54e-12	-2.99e-07
	Stand Dev	1.77e-06	2.37e-09	1.13e-06	9.77e-09	1.63e-07	1.15e-08	4.94e-09	8.50e-05
10 MPa	Min	-5.62e-07	-2.01e-08		-1.37e-08	-2.34e-07	-1.56e-08	-2.13e-08	-1.32e-04
to	Max	1.33e-06	1.98e-08		1.38e-08	2.79e-07	1.49e-08	2.48e-08	1.33e-04
16.53 MPa	Average	2.86e-07	6.11e-10		-9.75e-11	2.71e-09	7.90e-11	6.56e-11	-1.00e-06
	Stand Dev	4.29e-07	9.53e-09		6.16e-09	8.38e-08	6.69e-09	7.15e-09	5.93e-05
16.53 MPa	Min	-4.66e-07	-1.91e-08		-1.63e-03	-2.64e-06	-4.87e-03	-2.66e-06	-9.76e-05
to	Max	7.12e-07	1.89e-08		9.34e-09	2.43e-06	2.05e-07	4.01e-06	9.75e-05
22.064 MPa	Average	1.13e-07	-1.83e-10		-4.67e-06	-6.02e-09	-5.01e-06	4.05e-09	-7.67e-07
	Stand Dev	2.84e-07	9.43e-09		8.52e-05	2.11e-07	1.54e-04	2.40e-07	4.19e-05
22.064 MPa	Min	-3.08e-07	-1.80e-08						
to	Max	5.42e-07	1.71e-08						
100 MPa	Average	4.18e-08	9.19e-11						
	Stand Dev	1.18e-07	8.35e-09						

ASTEM97 Backward equations for $p(v,h)$, $T(v,h)$, $p(v,s)$ and $T(v,s)$

As shown in Figure 5, the saturated fluid specific density presents difficulty in developing the backward equations based on (v,h) and (v,s) for the very low pressure range of 611.213 Pa to about 1,080 Pa.

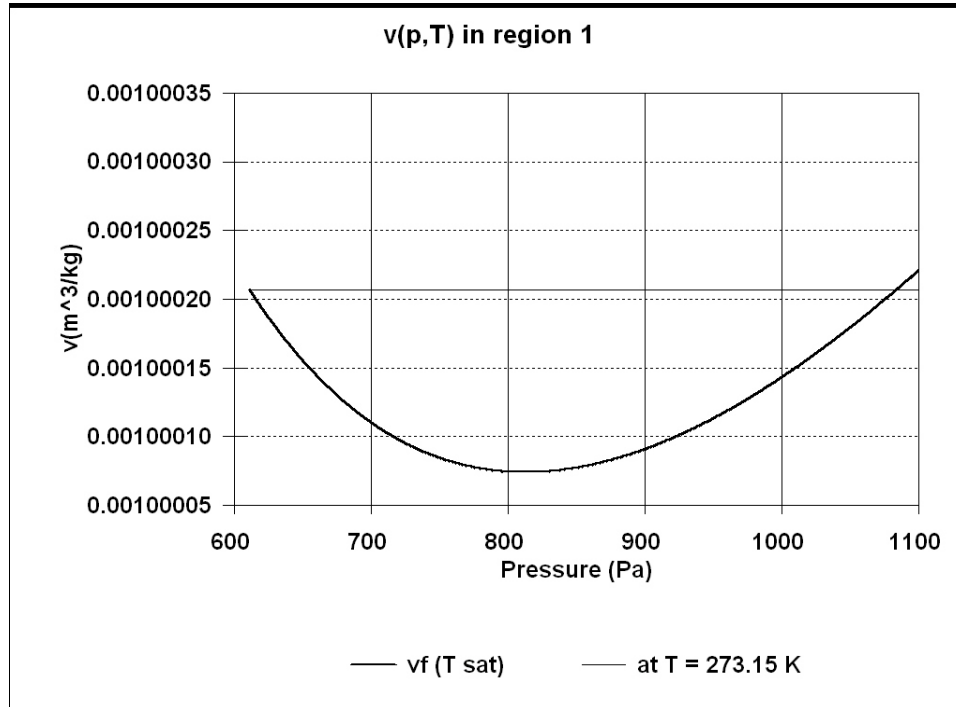


Figure 5 - Specific volume in Region 1 at low pressures

Tables 3 thru 6 show the pressure error in percent for the $p(v,h)$ and $p(v,s)$ backward equations. $\Delta T2$ in Tables 3 and 4 is 1.0×10^{-7} K. $\Delta T1$ in Tables 5 and 6 is 0.01 K.

Table 3 - $p(v,h)$ pressure error on 623.15 K and 23B boundaries in percent

Pressure	T (K)	623.15- $\Delta T2$	623.15	623.15+ $\Delta T2$	T23B- $\Delta T2$	T23B	T23B+ $\Delta T2$
Range							
16.53 MPa	Min	-4.05e-08	-3.24e-09	-1.21e-07	1.83e-07	-4.31e-03	-1.29e-08
	to	1.83e-07	3.18e-09	7.99e-08	4.88e-03	1.60e-08	6.30e-09
22.064 MPa	Average	8.11e-10	2.83e-11	2.54e-11	2.15e-03	-8.52e-06	-5.52e-10
	Stand Dev	1.51e-08	1.51e-09	2.30e-08	8.16e-04	1.90e-04	5.08e-09
22.064 MPa	Min	-3.89e-02	-5.40e-09	-6.80e-08	-9.19e-03	-1.05e-09	-9.97e-09
	to	1.76e-02	5.44e-09	5.70e-08	1.06e-02	1.02e-09	9.56e-09
100 MPa	Average	4.96e-05	-7.03e-12	-2.19e-11	-5.07e-04	-9.81e-13	8.22e-11
	Stand Dev	1.15e-02	2.70e-09	8.20e-09	4.55e-03	3.59e-10	3.55e-09

Table 4 - p(v,s) pressure error on 623.15 K and 23B boundaries in percent

Pressure	T (K)	623.15-ΔT2	623.15	623.15+ΔT2	T23B-ΔT2	T23B	T23B+ΔT2
Range							
16.53 MPa	Min	-1.16e-07	-3.24e-09	-1.56e-07	1.83e-07	-1.73e-09	-2.31e-08
to	Max	1.83e-07	3.18e-09	1.36e-07	5.91e-03	2.52e-08	1.29e-07
22.064 MPa	Average	2.53e-09	2.83e-11	-9.68e-09	2.61e-03	1.89e-10	-4.18e-09
	Stand Dev	4.21e-08	1.51e-09	4.36e-08	9.93e-04	1.87e-09	7.27e-09
22.064 MPa	Min	-5.98e-02	-5.40e-09	-9.65e-08	-1.30e-02	-1.05e-09	-2.31e-08
to	Max	1.69e-02	5.44e-09	9.94e-08	1.26e-02	1.02e-09	1.93e-08
100 MPa	Average	-5.37e-03	-7.03e-12	2.02e-09	-1.80e-03	-9.81e-13	3.53e-11
	Stand Dev	1.46e-02	2.70e-09	2.61e-08	5.35e-03	3.59e-10	9.29e-09

Table 5 - p(v,h) pressure error in percent

Pressure	T (K)	273.15	1073.15	2273.15	Tf	Tf-ΔT1	Tg	Tg-ΔT1	X=0.5
Range									
611.213 Pa	Min	-2.82e-03	-2.48e-08	-2.30e-08	-2.82e-03	-2.82e-03	-2.30e-08	-7.23e-08	-2.01e-06
to	Max	2.53e-03	2.02e-08	2.39e-08	4.63e-07	2.53e-03	2.39e-08	3.90e-08	2.03e-06
10000 Pa	Average	-5.31e-06	-7.57e-10	-2.30e-10	-6.20e-05	5.84e-06	-2.30e-10	-5.51e-11	1.89e-08
	Stand Dev	4.26e-04	4.76e-09	5.22e-09	3.13e-04	3.64e-04	5.22e-09	5.26e-09	1.02e-06
10000 Pa	Min	-9.11e-05	-8.79e-10	-8.79e-10	-8.49e-08	-4.16e-05	-8.79e-10	-3.38e-09	-1.98e-06
to	Max	1.33e-04	9.20e-10	9.20e-10	1.02e-07	1.20e-04	9.20e-10	3.76e-09	1.98e-06
1 MPa	Average	8.62e-08	1.60e-12	1.29e-12	3.98e-11	2.77e-07	1.08e-12	8.36e-11	-2.08e-08
	Stand Dev	9.81e-06	9.08e-11	9.09e-11	1.70e-08	6.08e-06	9.06e-11	1.27e-09	9.65e-07
1 MPa	Min	-1.64e-06	-2.06e-11	-2.06e-11	-1.46e-08	-6.40e-07	-4.03e-11	-5.90e-09	-1.79e-06
to	Max	1.76e-06	1.98e-11	1.98e-11	1.58e-08	8.69e-07	4.34e-11	6.24e-09	1.76e-06
10 MPa	Average	8.37e-10	6.20e-14	-9.38e-14	4.39e-11	1.66e-09	1.44e-13	4.49e-11	-3.72e-08
	Stand Dev	3.38e-07	7.12e-12	4.43e-12	3.95e-09	9.53e-08	1.51e-11	1.85e-09	8.25e-07
10 MPa	Min	-1.92e-07	-3.20e-11		-2.69e-09	-1.11e-07	-6.62e-11	-1.06e-08	-1.28e-06
to	Max	1.92e-07	3.16e-11		2.50e-09	1.06e-07	6.43e-11	1.43e-08	1.23e-06
16.53 MPa	Average	-2.24e-10	8.24e-14		-3.39e-11	3.87e-10	-1.07e-13	-1.55e-10	-3.85e-09
	Stand Dev	8.62e-08	1.46e-11		1.03e-09	3.50e-08	2.97e-11	3.76e-09	6.00e-07
16.53 MPa	Min	-1.22e-07	-4.33e-11		-1.85e-03	-2.64e-06	-5.06e-05	-2.66e-06	-2.38e-03
to	Max	1.22e-07	4.43e-11		9.61e-10	2.43e-06	3.02e-06	4.01e-06	1.06e-05
22.064 MPa	Average	-3.22e-10	2.70e-14		-5.66e-06	-6.14e-09	-6.99e-08	1.87e-09	-2.39e-06
	Stand Dev	6.03e-08	2.05e-11		9.40e-05	1.95e-07	1.75e-06	2.27e-07	7.51e-05
22.064 MPa	Min	-9.14e-08	-2.14e-10						
to	Max	8.70e-08	2.06e-10						
100 MPa	Average	7.78e-11	-3.97e-13						
	Stand Dev	2.68e-08	7.32e-11						

Table 6 - p(v,s) pressure error in percent

Pressure	T (K)	273.15	1073.15	2273.15	Tf	Tf-ΔT1	Tg	Tg-ΔT1	X=0.5
Range									
611.213 Pa	Min	-2.82e-03	-2.30e-08	-2.30e-08	-2.82e-03	-2.82e-03	-2.30e-08	-7.23e-08	-2.14e-06
to	Max	2.53e-03	2.39e-08	2.39e-08	1.37e-04	6.80e-02	2.39e-08	3.07e-08	2.15e-06
10000 Pa	Average	-5.31e-06	-2.30e-10	-2.30e-10	-5.82e-05	5.37e-02	-2.30e-10	-8.74e-11	1.51e-08
	Stand Dev	4.26e-04	5.22e-09	5.22e-09	3.09e-04	1.30e-02	5.22e-09	4.96e-09	1.09e-06
10000 Pa	Min	-9.11e-05	-8.79e-10	-8.79e-10	-8.49e-08	-6.33e-06	-8.79e-10	-9.65e-09	-2.11e-06
to	Max	1.33e-04	9.20e-10	9.20e-10	1.02e-07	5.11e-02	9.20e-10	9.74e-09	2.12e-06
1 MPa	Average	8.62e-08	1.60e-12	1.29e-12	3.98e-11	5.33e-03	1.08e-12	1.17e-10	-2.39e-08
	Stand Dev	9.81e-06	9.08e-11	9.09e-11	1.70e-08	1.34e-02	9.06e-11	3.85e-09	1.05e-06
1 MPa	Min	-1.64e-06	-2.06e-11	-2.06e-11	-1.46e-08	-6.91e-07	-4.03e-11	-1.29e-08	-1.96e-06
to	Max	1.76e-06	1.98e-11	1.98e-11	1.58e-08	1.02e-06	4.34e-11	1.33e-08	2.00e-06
10 MPa	Average	8.37e-10	6.20e-14	-9.38e-14	4.39e-11	-2.13e-09	1.44e-13	-4.54e-12	1.63e-08
	Stand Dev	3.38e-07	7.12e-12	4.43e-12	3.95e-09	1.63e-07	1.51e-11	4.94e-09	8.97e-07
10 MPa	Min	-1.92e-07	-3.20e-11		-2.69e-09	-2.34e-07	-6.62e-11	-2.13e-08	-1.34e-06
to	Max	1.92e-07	3.16e-11		2.50e-09	2.79e-07	6.43e-11	2.48e-08	1.33e-06
16.53 MPa	Average	-2.24e-10	8.24e-14		-3.39e-11	2.71e-09	-1.07e-13	6.56e-11	1.24e-08
	Stand Dev	8.62e-08	1.46e-11		1.03e-09	8.38e-08	2.97e-11	7.15e-09	6.25e-07
16.53 MPa	Min	-1.22e-07	-4.33e-11		-1.95e-03	-2.64e-06	-5.06e-05	-2.66e-06	-2.82e-03
to	Max	1.22e-07	4.43e-11		2.39e-05	2.43e-06	3.02e-06	4.01e-06	1.06e-05
22.064 MPa	Average	-3.22e-10	2.70e-14		-1.93e-06	-6.02e-09	-6.99e-08	4.05e-09	-2.84e-06
	Stand Dev	6.03e-08	2.05e-11		6.15e-05	2.11e-07	1.75e-06	2.40e-07	8.93e-05
22.064 MPa	Min	-9.14e-08	-2.14e-10						
to	Max	8.70e-08	2.06e-10						
100 MPa	Average	7.78e-11	-3.97e-13						
	Stand Dev	2.68e-08	7.32e-11						

ASTEM97 Backward equations on the saturation lines (x=0 or x=1)

In general, **ASTEM97** does reasonable well along the saturation lines. Figures 6 through 11 are provided to show those regions where **ASTEM97** does not predict the data set to be on the saturation line. The figures show the error in the pressure obtained when the quality is not predicted to be zero or one for p(h,s), p(v,h) and p(v,s). These occur at very low pressures, less than 1100 Pa, near the 623.15 K region, and for pressures between 19 MPa and 22.064 MPa.

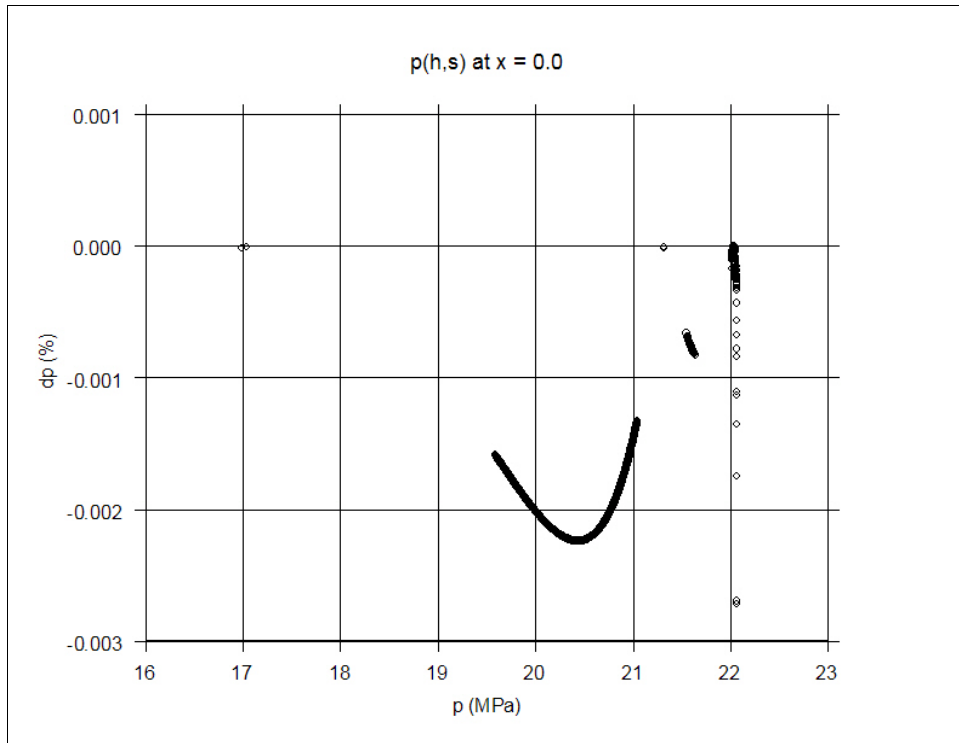


Figure 6 - Pressure error on p(h,s) on x = 0

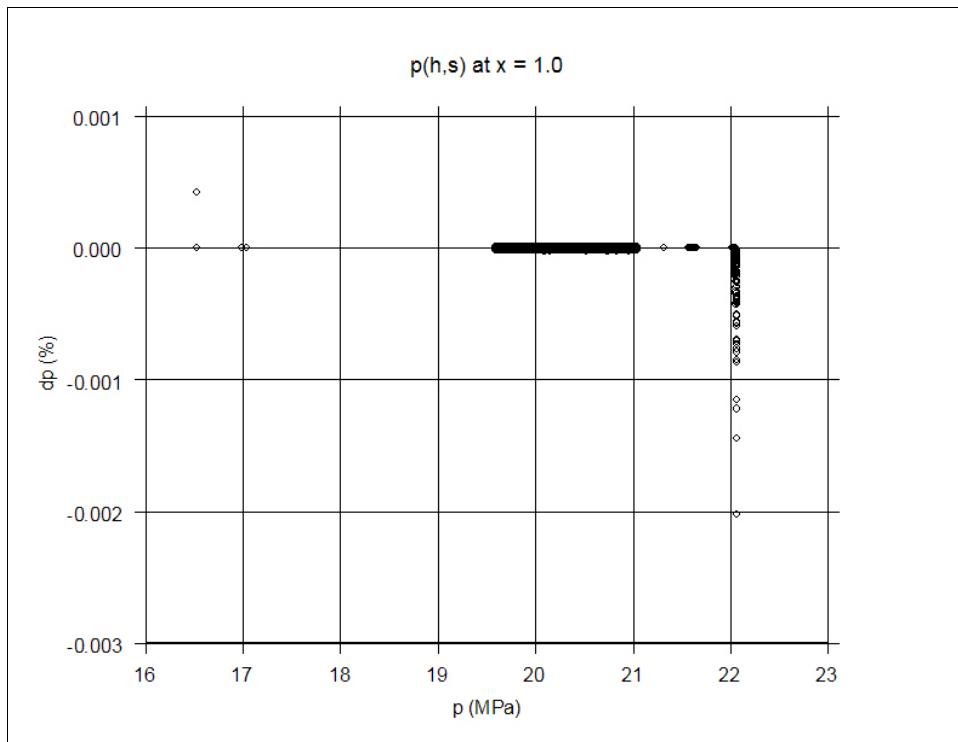


Figure 7 - Pressure error on p(h,s) on x = 1

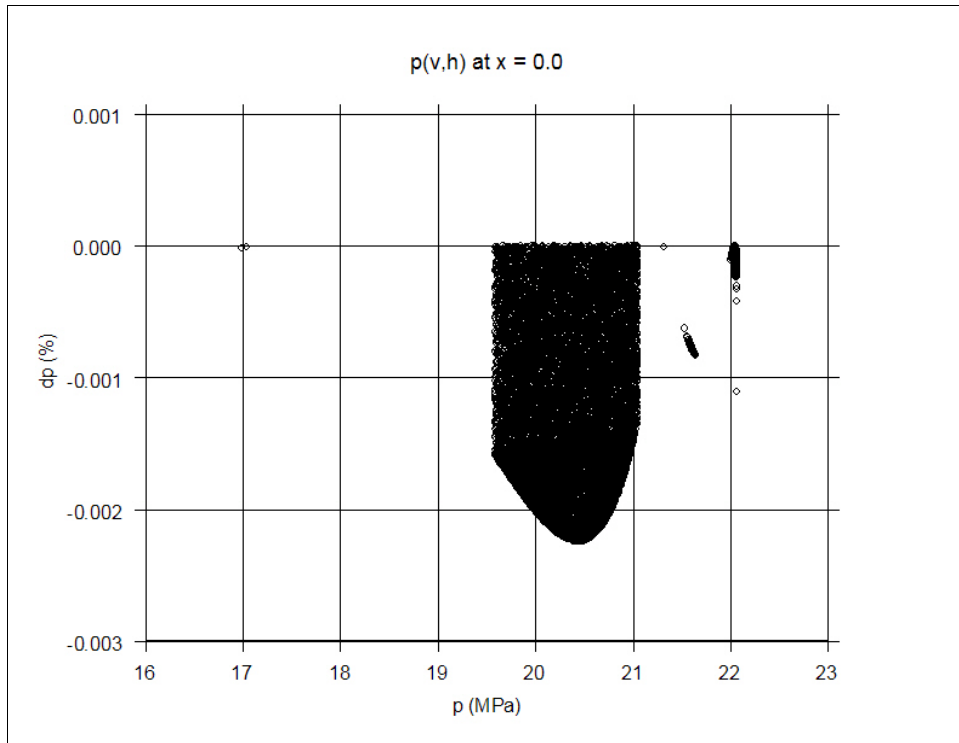


Figure 8 - Pressure error on p(v,h) on x = 0

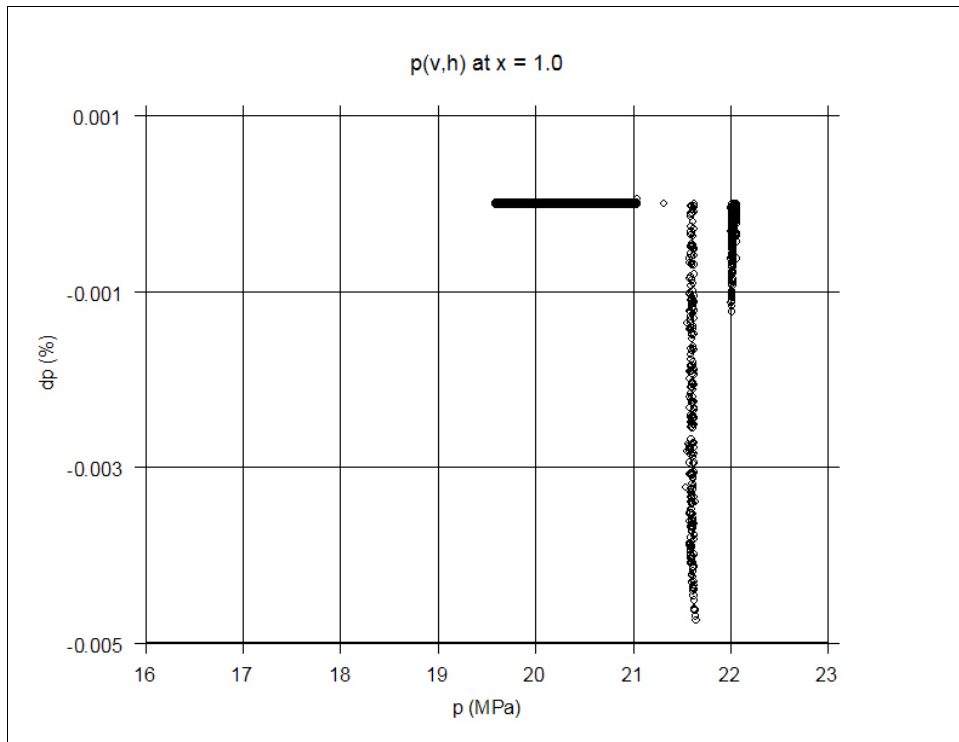


Figure 9 - Pressure error on p(v,h) on x = 1

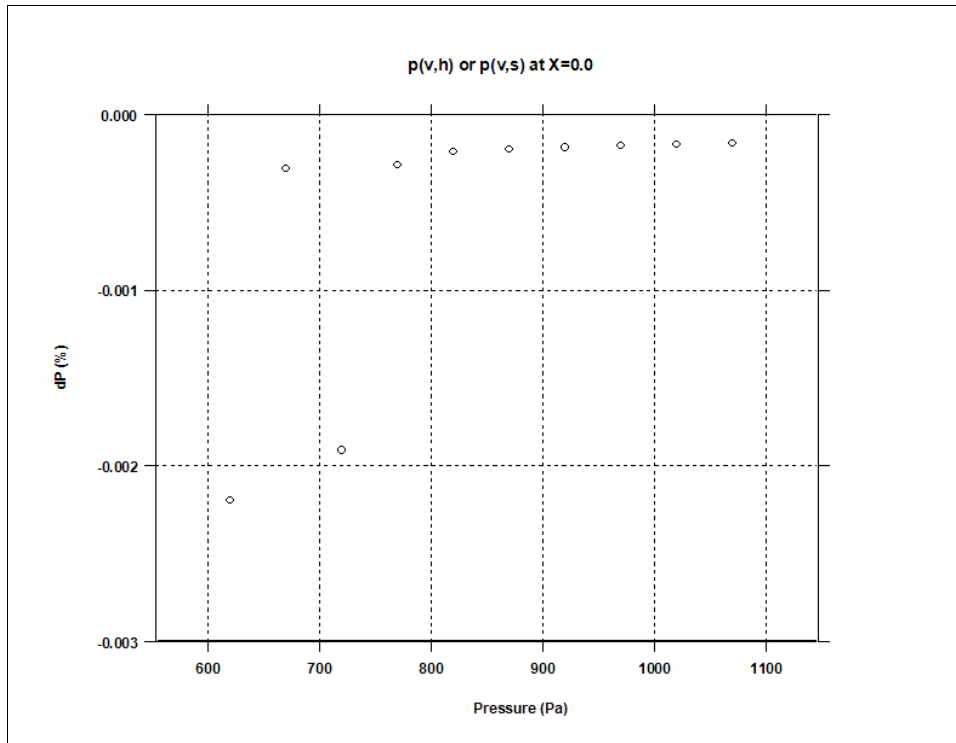


Figure 10 - Pressure error on p(v,h) or p(v,s) on x = 0

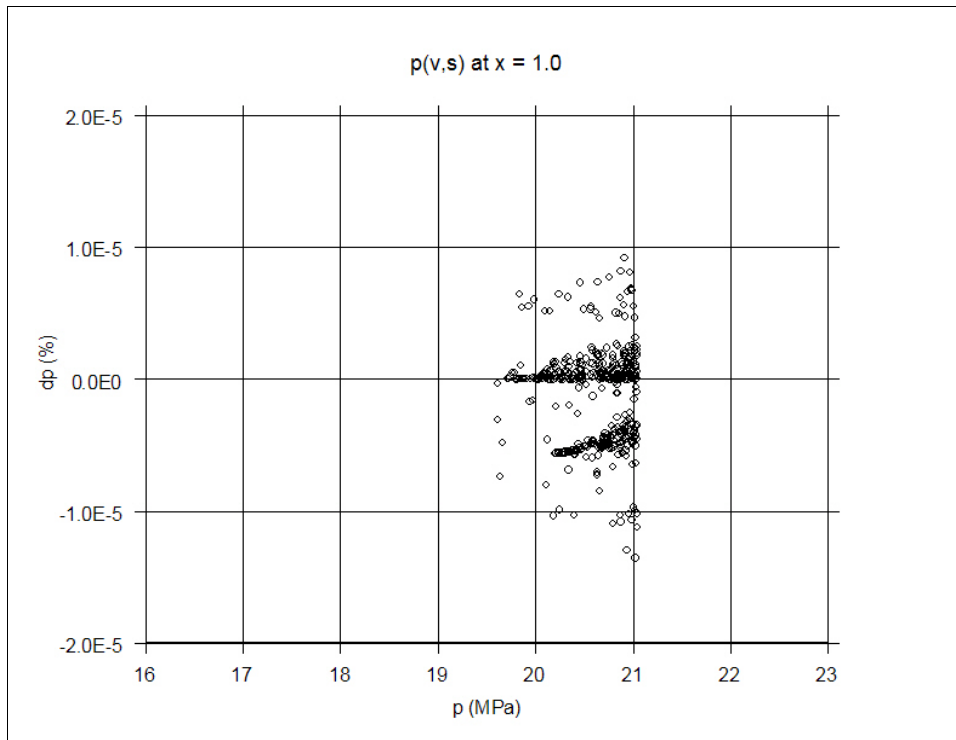


Figure 11 - Pressure error on p(v,s) on x = 1

ASTEM97 Backward Equation HSMAX97(s)

In developing the backward equation to obtain h based on s for the 100 MPa limit line, the discontinuity between IF97 Region 1 and Region 3 near 623.15 K and between IF97 Region 3 and Region 2 near 863.15 K is accounted for by using the maximum h value at s to define the (h,s) envelope — that is to place the point within the IF97 range. See Figures 12 and 13 for the 623.15 K region and Figures 14 and 15 for the 863.15 K region.

The overlapping s region near 623.15 K, where h from the IF97 Region 3 equation exceeds h from the Region 1 equation at 100 MPa, is from about 623.1500009 K to 623.15208 K. In **ASTEM97**, if the input s value is in the overlap range, the h estimate returned from HSMAX97 is increased by a small amount (0.00435 kJ/kg) to cover the discontinuity. In the overlapping region near 863.15 K, the h from the Region 2 equation exceeds the value from the Region 3 equation. Therefore no correction is needed.

For the overlap region near 623.15 K, the maximum relative error in using the estimated h from s to obtain the pressure at (h,s) is about 0.004%, comparing the return value from PHSBK1 and PHSBK3 (~3.85 kPa).

Summary

The values returned by the **ASTEM97** backward equations along any IF97 boundary, the minimum pressure or temperature limit, the maximum pressure or temperature limit, the saturation line or the boundaries between regions (1-3, 2-3, or 2-5), are subjective and based on the convergence criteria used. They are intended to be as reasonable as is practical. This also applied to the **ASTEM97** routines used to identify an IF97 region, as for all practical purposes these can be considered as backward equations as well.

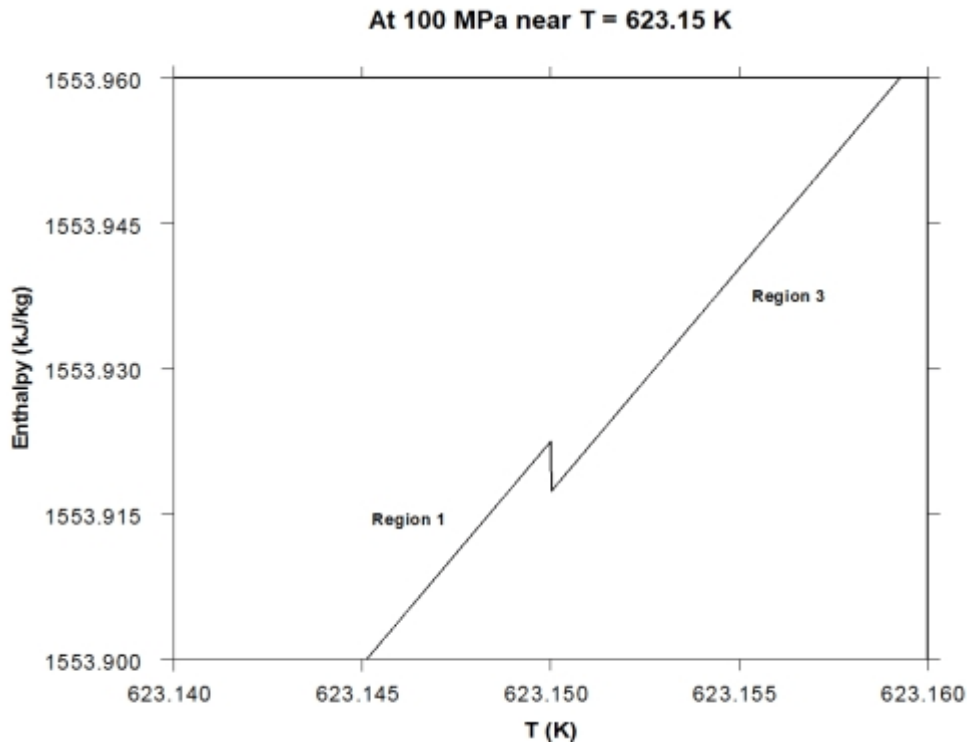


Figure 12 - (T,h) plot near 623.15 K

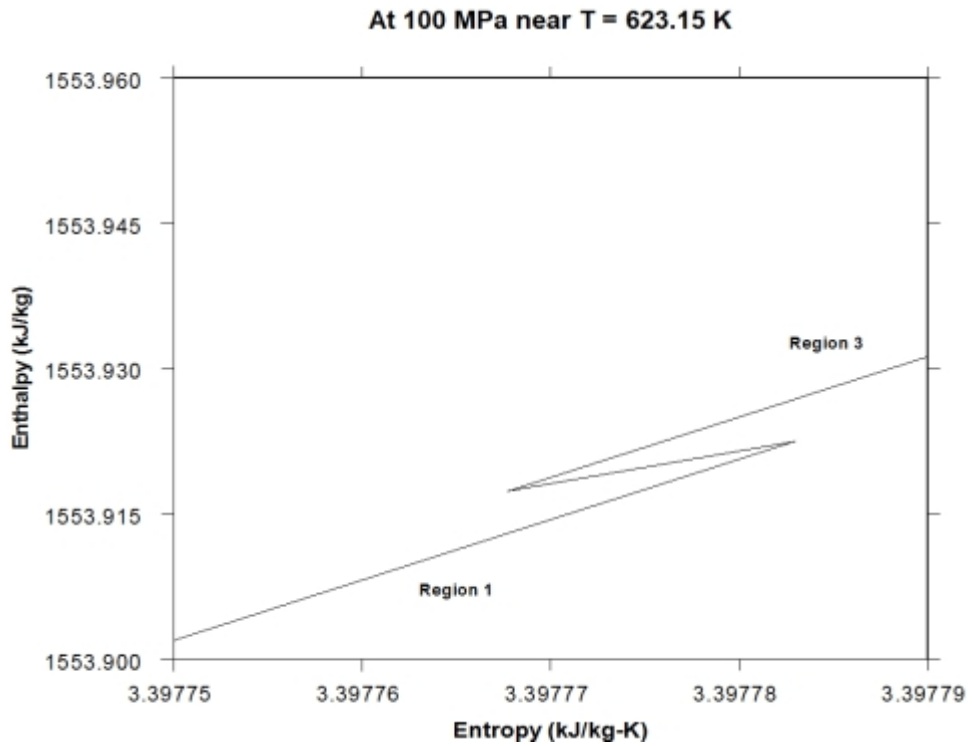


Figure 13 - (s,h) plot near 623.15 K

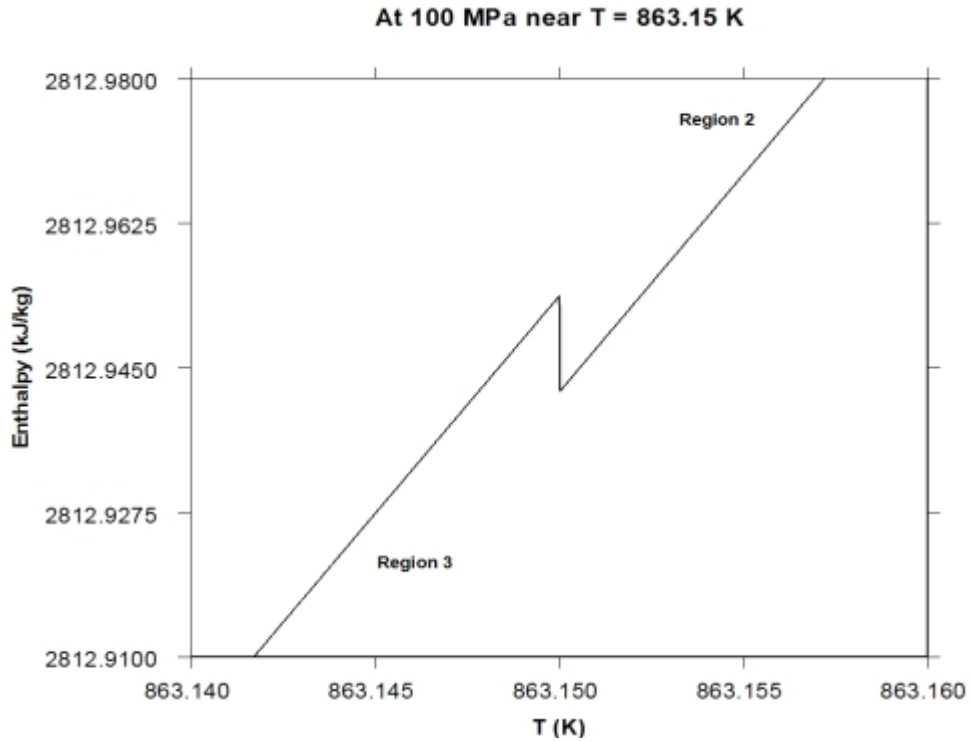


Figure 14 - (T,h) plot near 863.15 K

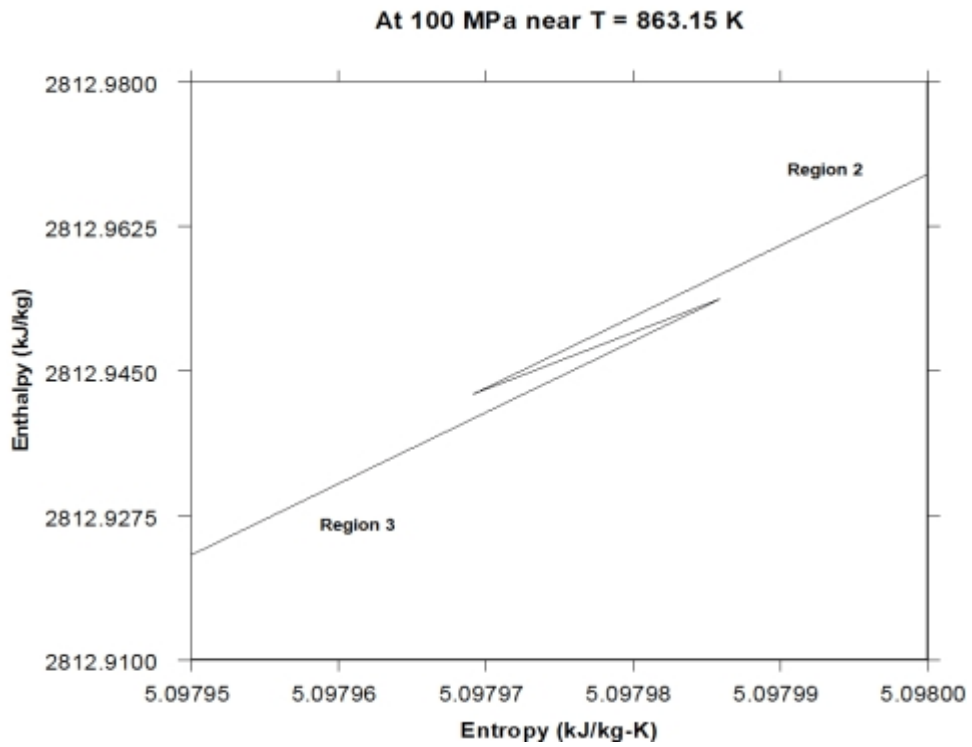


Figure 15 - (s,h) plot near 863.15 K

IAPWS Backward Equation for $v(p,T)$

A comparison of the specific volume for the saturated liquid and saturated vapor as calculated with VR3PT97B is compared to ROOT3MAX in Figures 16 and 17, to supplement the IAPWS release.

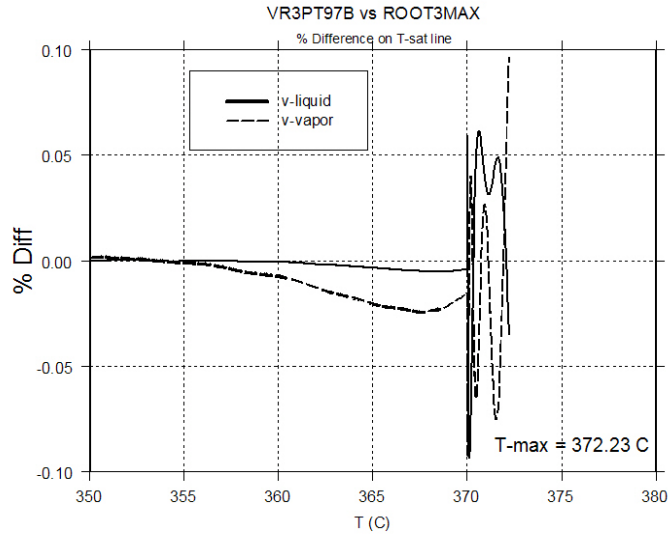


Figure 16 - VR3PT97B vs ROOT3MAX to 372.23 C

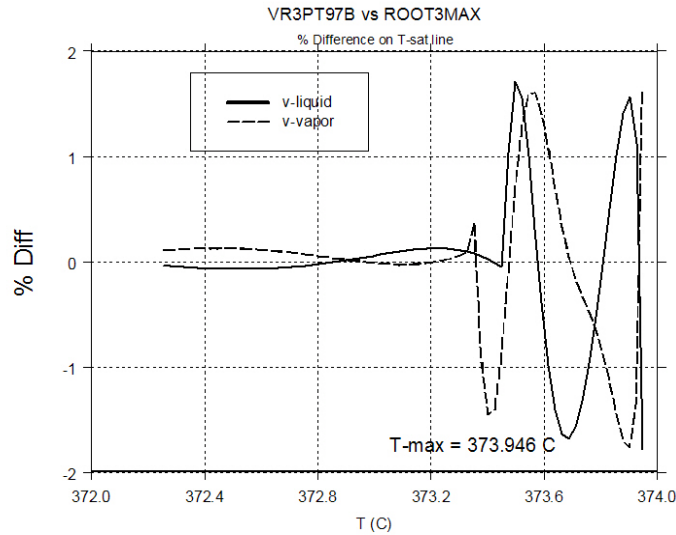


Figure 17 - VR3PT97B vs ROOT3MAX to 373.946 C

These comparisons are provided to assist a user in determining when VR3PT97B can be used to obtain the saturated specific volume to determining the density ($1/v$) for use in the density, temperature functions.